

Optimal Pseudo-Dynamic Planning of MV Radial Distribution Networks Considering Power Transit for Retail Market Trading

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Abstract

The paper tries to present an improved mathematical model for distribution companies (DISCOs) in order to solve Distribution Network Expansion Planning (DNEP) problems in the presence of certain resources of Distributed Generations (DGs) entitled small scale Gas Engines in the medium voltage grids. With the aim of minimizing investment and operation costs, the recommended model combines a comprehensive multi-objective optimization model with grid designer's experience and power transit possibility in retail trading. The proposed model determines the optimal capacity of new transformers, and the most efficient type and route of the expandable medium voltage feeders under probable power transit condition in retail markets. In addition, this model maintains system reliability along with an innovative formulation. The scenario-based results obtained after implementing this method on a typical distribution grid modeled under GAMS software platform indicate that the proposed model is really suitable for expanding distribution grids.

1. Introduction

Distribution network expansion planning (DNEP) seeks to determine the minimum cost design that ensures adequate substation and line capacity to meet forecasted load over the planning horizon. The expansion plan specifies additional line routes, the conductor types, the substations to be reinforced and those to be constructed. DNEP is aimed at minimizing equipment and network operational costs while satisfying technical constraints such as load bus voltage limits, line current-carrying capacity, and the network acyclic structure. The expansion plan can represent the planning horizon using either a single or multiple stages [1].

Generally, the network planning can be transmission-related or distribution-related. Transmission network planning mainly deals with upgrading existing circuits or adding new in order to satisfy power transfer requirements while meeting security constraints [2]. The distribution network planning deals with sitting and sizing distribution substations and feeders. As mentioned in [3], both transmission and distribution networks comprise of lines/cables, substations and generations. However, due to specific characteristic of a distribution system

(such as its radial characteristics), its planning is normally separated from a transmission system, although much of the ideas may be similar.

Distributed generation (DG) systems are considered an integral part in future distribution system planning. The active and reactive power injections from DG units, typically installed close to load centers, are seen as a cost-effective solution for distribution system voltage support, energy saving, active power loss reduction, and network investment deferral.

Currently, distribution systems are transferring from being passive networks to be active ones by allowing installation of DG units. Dynamic planning of active distribution networks needs an effective and intelligent algorithm to consider the integration of DG and demand variation although still satisfying the operational constraints of the system.

This type of planning determines where, when and what types of components must be installed and/or constructed within the planning period in order to meet the needs of the electricity distribution services, with technical and operational specifications regarding quality of the supply service, as well as seeking the lowest possible operation and investment costs. After distribution substations and primary feeders have been routed, the system needs to be designed in detail. It is the point at which reliability concerns become more important [4].

Pseudo-dynamic expansion planning is carried out in several stages to determine the capacity of reinforcements and the stage and places in which they should be installed to meet the growing demand with minimal cost and acceptable quality standards. Conventional alternatives for expansion are rewiring, network reconfiguration, and installation of new feeders, capacity expansion and construction of new substations [5].

In restructured power systems, DISCOs (Distribution Companies) aim to improve their profits and minimize the investment risk to meet the growth demands in their territories while keeping their customer's bills affordable. DISCO planners venture to implement new planning strategies for their network in order to meet the load growth economically, serve their customers with a reliable electricity supply, and survive in the competitive electricity market [6].

These goals can be achieved by introducing new alternatives for solving the DNEP problem in addition

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to the traditional options which make it difficult due to creation of many decision making variables. On the other side, the high volume of investment and energy and equipment prices, the economic risk especially in restructured environment have increased so that network planners introduce more accurate and appropriate methods for designing these networks.

The DG units considered in this paper are the Gas Engine units capable of operating at base load to provide committed generation.

A. Literature Review

The aim of the most of works carried out so far is to find a planning to expand the system by analyzing the impacts of the construction and/or upgrading of substations, constructing new feeders and eventually changing the size of existing feeder conductors considering the predetermined planning scope.

A cost-benefit analysis-based heuristic approach with the consideration of multiple load levels and fluctuations in electricity market price has been proposed in [6] to minimize the investments with DG integration. In [7], an optimization model is proposed for distribution system expansion with DG in order to minimize the total cost over a planning period. In [8], the impact of increasing DG penetration on system losses has been analyzed for different generating resources. An analytical approach is developed in [9] in order to minimize energy losses by optimal DG placement in a distribution system. A methodology based on genetic algorithm (GA) is presented in [10] to minimize network losses with the consideration of system constraints such as reliability, voltage limits, and DG penetration. An iterative method based on analysis of system voltage stability is proposed in [11] for optimal placement of DG units in distribution networks. The impact of network investment deferral on DG expansion is analyzed in [12] by considering DG at various candidate locations.

In [13], a stochastic dynamic multi-objective model is proposed for integration of distributed generations in distribution networks. The proposed model optimizes three objectives, namely technical constraint dissatisfaction, costs and environmental emissions and simultaneously determines the optimal location, size and timing of investment for both DG units and network components.

The proposed model in [14], simultaneously optimizes two objectives, namely the benefits of DISCO and DG-Owner and determines the optimal schemes of sizing, placement and specially the dynamics (i.e. timing) of investments on DG units and network reinforcements over the planning period.

Finally, [15] recognizes the importance of the radiality constraints but provides no detailed analysis of the subject.

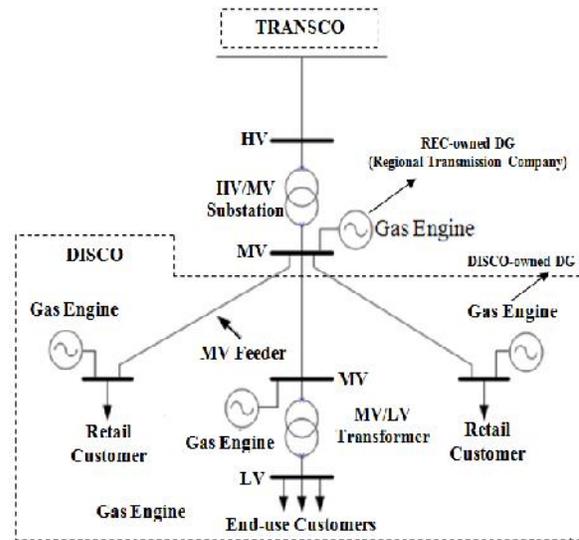


Fig. 1 Proposed locations for installation of small-scale gas engines (dispatchable DGs).

B. Motivation and Approach

A typical configuration of a DISCO-TRANSCO layout is shown in Fig.1, where a DISCO is receiving power through its junction substation connecting it to the main grid, for meeting its customer demand in the distribution network.

Fig.1 also illustrates the possible locations of gas engines when the gas network has been established near the electric network and can provide considerable opportunities for DISCOs or private investors to install and operate dispatchable DGs.

The paper proposes the implementation of one of the dispatchable DGs, namely Gas Engine as an economical attractive new tool for DNEP problem in MV grids to benefit DISCOs by Justing of their economical and operational advantages, solve the lacking electric power supply problem, Justing and meet the load growth requirements with a reasonable price.

The new DNEP model with Gas Engine implementation aims to minimize DISCO's capital investment and operating costs in new facility capacities (substations, transformers and feeders) as well as payments to be made toward DISCO's system loss compensation, cost of DG's generated power, and cost of power purchased from the electricity market.

C. Contributions

From the above literature review, it seems clear that the explicit representation of the radiality constraints is an issue that has not yet been appropriately solved.

Within this context, the main contributions of this paper are fourfold:

1) A mathematical model is proposed from DISCOs' point of view for the DNEP problem in

which the radiality constraints are represented explicitly using graph theory.

2) Power Transit modeling as an essential constraint for DNEP toward implementation of power trading and bilateral contracts in Retail Markets.

3) This model is formulated as a mixed-integer-nonlinear programming on GAMS platform using binary decision variables, which gives accurate decisions, where unity or zero decision variables mean, invest or do not, respectively. Hence, DISCO can make optimal decisions without the need to round the final decision variables that usually occurs in some traditional methods.

4) By presenting a modified multi-objective function and new constraints governing the conditions of the problem, the recommended model aims at a practical study and economization of investment in the private sector through small scale Gas Engines (dispatchable DGs) and shows its impact on expansion planning of medium voltage distribution networks.

Meanwhile, this model is solved easily using CPLEX 11 [16] under GAMS [17] on a Windows-based PC with a processor clocking at 3.08 GHz and 1 GB of RAM.

2. Proposed Model Explanation

A. Load Forecasting

In this formulation, it has been assumed that DISCOs own the distribution system and operate it and are also responsible for supplying energy for their customers. Moreover, in order to load modeling, the amount of MV/LV substation loading at each bus is equal to the aggregated values of all load points connected to that bus.

B. DNEP Mathematical Model Formulation

Distribution system planning is relatively complex task that investigates many technical and economical issues based on some constraints such as maximum voltage drop at load buses, loading capacity of substations and DGs, radial structure of distribution networks.

The objective function of DNEP problem includes six cost components which have been mathematically formulated in the following equations:

$$\text{Min } C_{\text{Total}} = C_{\text{Fixed}} + C_{\text{Variable}} \quad (1)$$

that is,

$$C_{\text{Fixed}} = C_{\text{Feeder}} + C_{\text{Trans}} \quad (2)$$

$$C_{\text{Variable}} = C_{\text{Operation}}^{\text{SS}} + C_{\text{Operation}}^{\text{GE}} + C_{\text{Reliability}} + C_{\text{Loss}} \quad (3)$$

The objective function consists of total selected MV feeders cost, total selected MV transformers cost, total operation cost of HV/MV substation, total operation cost of Mini Gas Engine Units, total cost of supply interruption, and total cost of electrical losses, respectively.

MV Feeders' Cost

Cost of constructing new MV feeders has been modeled as follows:

$$C_{\text{Feeder}} = \sum_{i=1}^{n_t} \sum_{j=1}^{n_l} C_{f,ij} \cdot L_{f,ij} \cdot ij \quad (4)$$

Transformers' Cost

Cost of transformers provision which have been proposed for expansion of HV/MV substations is modeled as follows:

$$C_{\text{Trans}} = \sum_{u=1}^{n_u} C_{tr,u} \cdot u_{t,h} \quad (5)$$

$u_{t,h}$ is an integer decision-making variable which determines the optimal type and number of transformers with regard to the cost of installing them by the end of planning horizon year.

HV/MV Substation Operation Cost

In a restructured environment, in order to provide the required electrical power for their customers, DISCOs have to buy energy from electricity market and import it from main grid (upward grid) and sell it to their customers (downward grid).

$$C_{\text{Operation}}^{\text{SS}} = \text{NPW}(\quad) \quad (6)$$

$$\sum_{i=1}^{n_t} \sum_{j=1}^{n_l} \sum_{t=1}^{t_h} (V_{ss,t} \cdot (V_{ss,t} - V_{jt}) \cdot y_{ss,j}) \cdot \text{pf}_{\text{sys}} \cdot (8760 \cdot \text{LF}) \cdot \text{MCP}$$

Where from [18] and [19],

$$\text{NPW}(t) = \sum_{t=1}^{t_h} \frac{(1 + \text{InfR})^t}{(1 + \text{IntR})^t} \times t$$

Gas Engines Operation Cost

Operation cost of small scale gas engines is due to the cost of fuel and their periodic maintenance (PM). In the recommended model, it has been assumed that GE units have been installed on load buses. Thus:

$$C_{\text{Operation}}^{\text{GE}} = \text{NPW}(\quad) \quad (7)$$

$$\sum_{j=1}^{n_l} \sum_{t=1}^{t_h} (S_{\text{GE}_{jt}} \cdot \text{pf}_{\text{GE}}) \cdot (8760 \cdot \text{LF}) \cdot C_e^{\text{GE}}$$

Cost of Supply Interruption

In radial networks there are no alternative supply routes and the outage of a branch interrupts the delivery to all consumers supplied through this branch. Thus, the cost of supply interruption can be easily calculated using this expression:

$$C_{\text{Reliability}} = \text{NPW}(\quad) \quad (8)$$

$$\sum_{i=1}^{n_t} \sum_{j=1}^{n_l} \sum_{t=1}^{t_h} \left(\sqrt{3} \cdot V_n \cdot \text{Re} \{ I_{fij} \} \right) \cdot \left(r_{fij} \cdot r_{fij} \right) \cdot \text{LF} \cdot C_{\text{ENS}}$$

Cost of Network Electrical Loss

The Cost of energy losses equals

$$C_{\text{Loss}} = \text{NPW}(\quad) \quad (9)$$

$$\sum_{i=1}^{n_t} \sum_{j=1}^{n_l} \sum_{t=1}^{t_h} (V_{i,t} - V_{j,t})^2 \cdot y_{ij} \cdot \text{pf}_{\text{sys}} \cdot (8760 \cdot \text{LF}) \cdot \text{MCP}$$

The objective function in (1) is minimized subject to various constraints, which range from system to facilities constraints.

The DNEP problem constraints:

1. Total Power Conservation:

The summation of all incoming and outgoing power over the DISCO's feeders, taking into consideration the DISCO's feeders losses and the power supplied by DG, if it exists, should be equal to the total demand at that bus.

$$\sum_{i=1}^{n_t} (S_{ij,t} - (V_{ij,t})^2 \cdot y_{ij}) + S_{GE_{j,t}} = S_{D_{j,t}} \quad (10)$$

2. Maximum Permissible Voltage Drop:

The variation of voltage drop of DISCO's load points should be in a permissible range.

$$-V_{max} \leq V_{ij,t} \leq V_{max} \quad (11)$$

3. Substation Expansion Capability:

The proposed optimization model in order to determine the necessity of expansion of HV/MV substation by adding transformers during the planning years, has implemented a constraint that this optimization problem converges effortlessly to its optimum point.

$$\sum_{j=1}^{n_l} S_{ss,j,t} \leq ss \cdot \sum_{u=1}^{n_u} S_{tr,u} \cdot u_t \quad (12)$$

4. Distribution feeder's Thermal Capacity:

The DISCO's feeder has a capacity limitation for the total power flow through it during peak loads.

$$S_{ij,t} \leq S_{ij,max} \quad (13)$$

5. Power Transit Capability

A retail electricity market exists when end-use customers are able to choose their supplier from various electricity retailers.

In economic terms, electricity (both power and energy) is a commodity capable of being bought, sold and traded.

A powerful retail electricity market should be supported and implemented by proper trading tools that take into consideration special circumstances of electricity trading which are different from other commodity trading practices.

A successful implementation of a trading system in electric energy could fulfill restructuring objectives, which include competition and customer choice, and serve vital needs of electricity market participants. Therefore, *Open Access* for distribution networks is essential to have a thoroughly competitive retail market.

In this paper, a minor modification in power flow equations has been applied in order to consider power transit capability of MV networks as a constraint for DNEP problem.

If we consider bus j as an 'injection point' into MV network then the equation (10) should be modified as

constraint (14) and if this bus is taken into account as a 'delivery point' then its power flow equation have to be replaced by constraint (15) as follows.

$$\sum_{i=1}^{n_t} (S_{ij,t} - (V_{ij,t})^2 \cdot y_{ij}) + S_{GE_{j,t}} = S_{D_{j,t}} - S_{Transit} \quad (14)$$

$$\sum_{i=1}^{n_t} (S_{ij,t} - (V_{ij,t})^2 \cdot y_{ij}) + S_{GE_{j,t}} = S_{D_{j,t}} + S_{Transit} \quad (15)$$

6. Radial and Network Connectivity Check

A network graph is connected if there is a path between any two of its nodes. For a distribution network it means that all load nodes are connected to the source node and can be supplied from this node.

To investigate radial topology of distribution networks, an appropriate and practical algorithm for determining and checking of radial configuration in each stage of planning has been presented in this paper. Here, this constraint is verified by MATLAB codes apart from other constraints which have been implemented under GAMS platform.

If a distribution system is modeled by a graph, because of its radiality topology it will be undoubtedly a tree. Hence, it is essential to find all the possible trees during expansion years.

A graph is a tree if and only if it satisfies both of the following conditions:

- 1- It must have no cycles (no mesh).
- 2- It must be connected (a spanning tree).

Let $G=(V,E)$ be a graph in which the elements of V are vertices (or nodes) and the elements of E are edges (or lines).

The number of cycles of the graph G equals:

$$N_{mesh} = V - E + 1 \quad (16)$$

The first condition is satisfied if $N_{mesh} = 0$.

The second condition (graph connectivity) can be simply checked using the following 5-step proposed algorithm:

Step1) Determine [A] as an Adjacency Matrix for the graph G.

Step2) Define [] as a diagonal matrix which the value of each element $_{ij}$ equals the degree of i^{th} node of the graph G.

Step3) Define Laplacian [Q] as $Q \equiv \Delta - A$

Step4) Calculate the Eigenvalues of [Q] and put them in order as follows

$$\sim_0 (= 0) \leq \sim_1 \leq \sim_2 \leq \dots \leq \sim_{n-1}$$

Then, the graph G is connected if $\sim_1 > 0$

Step5) If [J] is a Identity Matrix with the same order as [A], then the number of trees will be $\frac{\det(J+Q)}{n^2}$

To indicate the capability of above-mentioned algorithm, a typical example is shown in Figure 2.

Table 2 Cost and capacity of candidate transformers

Type	Capacity (MVA)	Cost (\$)
Candidate 1	15	430000
Candidate 2	30	700000

Table 3 Cost and thermal capacity of candidate MV feeders

Type	Thermal Capacity (MVA)	Cost (\$/Km)
1	5	10000
2	10	18000
3	20	25000

It is observed that to solve the proposed DNEP problem, the detailed values of forecasted loads and the potential capacity of dispatchable DGs are the initial and crucial input data. DISCO is responsible to provide them in advance in order to obtain a credible expansion plan (see Table 1 and Table 5).

It should also be approved that the line capacity and the type are determined based on capacity, voltage level, and the type of supplying substation.

As noted earlier, The optimal routing of MV feeders for each scenario and each stage has been

obtained through GAMS and MATLAB interfacing output (see Fig. 3).

Table 4 Technical and economic data for expansion plan

Parameter	Unit	Value
V_n	KV	20
V_{max}	%	5
MCP	\$/MWh	40
C_e^{GE}	\$/MWh	30
C_{ENS}	\$/MWh	5800
InfR	% (biennial)	5
IntR	% (biennial)	8
pf_{sys}	-	0.85
pf_{GE}	-	0.9
LF	-	0.65
r_{ij}	hour	3
f_{ij}	failure/Km.year	0.2

Table 5 Capacity of dispatchable DG sources submitted beforehand by DISCO as input data

Year	Capacity (MVA)							
	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5	Bus 6	Bus 7	Bus 8
1 st	0	0	0	0	0	0	0	3
2 nd	0	3	0	0	0	0	0	3
3 rd	2	3	0	0	0	0	0	3
4 th	2	3	0	0	0	2	0	3
5 th	2	3	0	1	0	2	0	3
6 th	3	3	0	1	0	2	0	4
7 th	3	4	0	1	0	2	0	4
8 th	3	4	0	2	0	2	0	5
9 th	3	4	0	2	0	3	0	5
10 th	3	4	0	2	0	4	0	5

Table 6 The optimal selection of HV/MV transformers over expansion planning years

Scenario	Type	Number	Installation and Operation Time
A	Existing	-	All the time
	Candidate1	0	-
	Candidate2	1	From 3 rd year up to 10 th year
B	Existing	-	All the time
	Candidate1	1	From 8 th year up to 10 th year
	Candidate2	0	-

In contrast to renewable DGs such as wind turbines that are intermittent resources and their sitting has some environmental restrictions, this paper adopted the use of the natural gas generator set since this technology is known to be environmentally friendly and produces the least pollution compared to other fossil fuels DGs [7]. As a result, these

dispatchable DGs such as CHP and Gas Reciprocating Engines, can commit in peak load hours based on some earlier long term contracts in order to assist DISCO to meet a proportion of peak load at reasonable prices.

According to the results indicated evidently in Table 6, the commitment of dispatchable DGs (under

the ownership or operation of DISCO) has explicitly influenced over the selection of candidate transformers for expansion of HV/MV substation. Obviously, the transformer introduced by Scenario-B (Candidate 1) is smaller and also cheaper than its counterpart (Candidate 2) in Scenario-B.

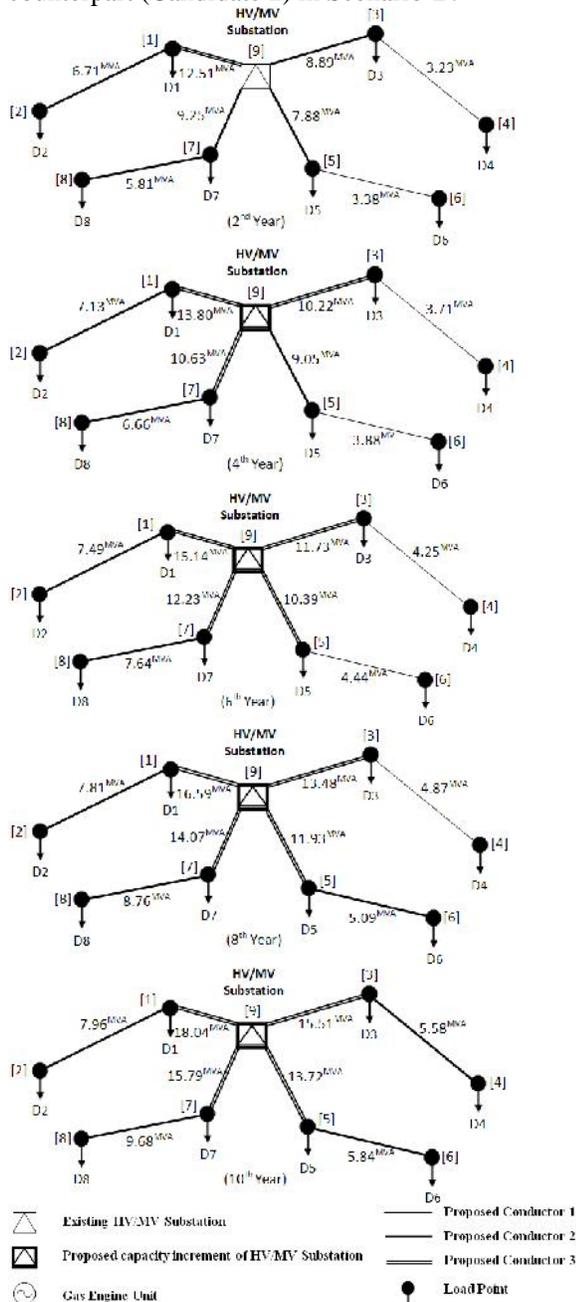
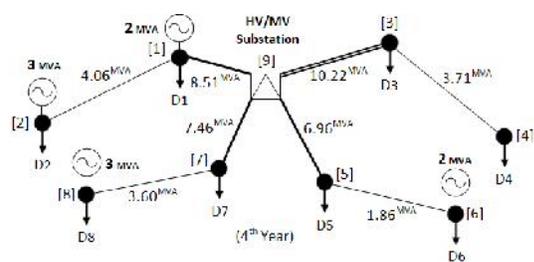


Fig. 5 Proposed radial structure in Scenario-A for each 2-year stage



The results obtained from the simulation of Scenario-A and Scenario-B have been illustrated in Fig. 5 and Fig. 6, respectively.

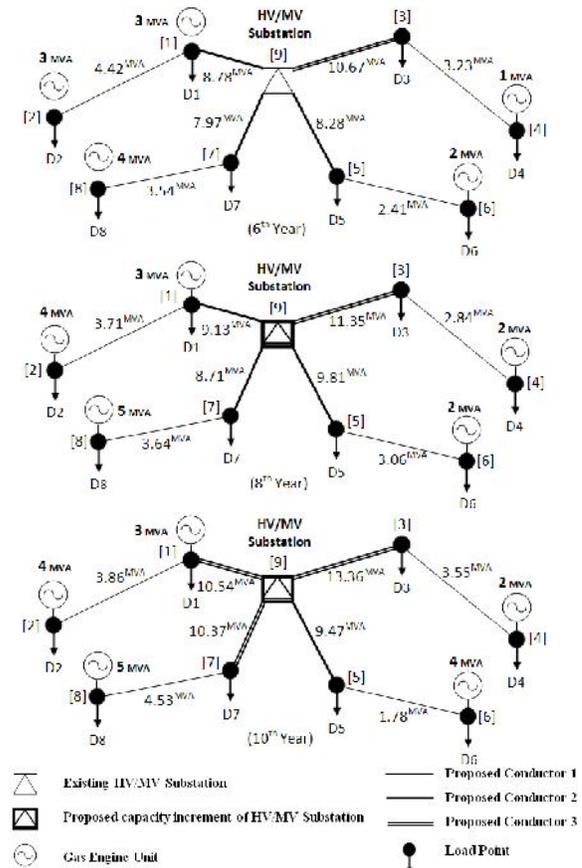


Fig. 6 Proposed radial structure in Scenario-B for each 2-year stage.

TABLE 7 COMPARISON OF FINAL SOLUTIONS BETWEEN SCENARIO A AND SCENARIO B

Cost Components	Scenario-A	Scenario-B
Primary Feeders Cost	2.119	1.820
MV Transformers Cost	0.7	0.43
HV/MV Substation	79.838	59.994
Operation Cost	0	14.828
DG Operation Cost	1.175	0.822
Supply Interruption Cost	2.885	1.385
Electrical Losses Cost	86.716	79.279
Total Cost (M\$)	86.716	79.279

Table 7 integrally demonstrates detailed results obtained from the optimization model. It is obvious that by investing in dispatchable DG rather than purchasing power at high market price rates from HV/MV substation, the DISCO can minimize its overall planning cost and reduce its customers' bills. This adds more social economical benefit to the use of dispatchable DGs as a new attractive tool in solving

the DNEP problem, rather than the traditional planning options.

Examining the implementation of DG as a key element in the DNEP is not only a matter of minimizing the total planning cost but also has social economic benefits as discussed above and electrical operational benefits.

To evaluate the impact of power transit capability of MV networks, the implementation results of this feature has been compared in two other scenarios with 2MVA transit of power from bus 2 to bus 4.

▪ **Scenario-C:**

To show the power transit capability without penetration of dispatchable DG units.

▪ **Scenario-D:**

To show the power transit capability in the presence of dispatchable DG units.

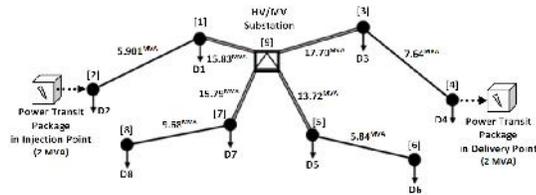


Fig. 7 Optimal network configuration at horizon year for Scenario C.

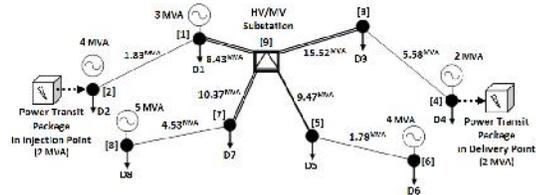


Fig. 8 Optimal network configuration at horizon year for Scenario D.

Table 8 Comparison of final solutions between Scenario C and Scenario D

Cost Components	Scenario-C	Scenario-D
Primary Feeders Cost	2.119	1.844
MV Transformers Cost	0.7	0.43
HV/MV Substation	86.358	59.978
Operation Cost		
DG Operation Cost	0	19.577
Supply Interruption Cost	0.814	0.509
Electrical Losses Cost	3.826	1.643
Total Cost (M\$)	93.818	83.981

In comparison with the results demonstrated in Table 7, although the total cost of planning in Scenarios C and D is slightly higher than Scenarios A and B, but in this way, the DISCO can keep its customers and reduce the risk of losing them by contracting with other DISCOs using its power transit capability which was considered as a flexible risk management method beforehand.

Meanwhile, the possibility of transit power can provide an open access environment for retail trading activities in the future competitive retail electricity markets based on smart grid goals.

4. Conclusion

In this paper, the problem of the expansion and reinforcement of radial distribution systems considering different scenarios and using dispatchable DG units, in particular small scale gas reciprocating engines, in parallel with traditional means such as cables and transformers, has been studied. The method takes into account the total annual cost including capital recovery, energy loss and undelivered energy costs. The approach presented has been applied for optimal new network planning. This paper presented a practical algorithm to incorporate the radiality constraints into active distribution system optimization problems more simply and efficiently. It also presented a preliminary analysis of the generalization of power transit capability as a constraint for planning issues.

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Nomenclature

- i Number of distribution system buses.
- j Number of system load buses.
- NPW (.) Net Present Worth function.
- LF Load factor.
- n_t Total number of distribution system buses.
- n_l Total number of system load buses.
- MCP Electricity market price (\$/MWh).
- C_e^{GE} Mini Gas Engine Units operating cost (\$/MWh).
- C_{ENS} Cost of energy not delivered (\$/MWh).
- $S_{D,j}$ Load demand at bus j (MVA).
- $S_{GE,j}$ Power generated from mini gas engines (MVA).
- $S_{tr,u}$ Capacity of candidate transformer u (MVA).
- S_{ij} Power flow in feeder connecting bus i to bus j (MVA).
- S_{ij}^{\max} Feeder’s thermal capacity limit from i to bus j (MVA).
- $S_{Transit}$ Power Transit as a commodity (MVA).
- $\{_{ij}$ Feeder i to j binary decision variable.
- \dots_{ss} Maximum substation capacity factor.
- $C_{f_{ij}}$ Feeder cost from i to j (\$/Km).
- $L_{f_{ij}}$ Feeder’s length from i to j (Km).
- u Type of candidate MV transformer.
- n_u Set of candidate MV transformers.
- $C_{tr,u}$ Candidate transformer u cost (\$).
- \mathbb{E}_{u,t_h} Candidate transformer u integer decision variable.
- y_{ij} Feeder segment admittance from bus i to bus j represented by Y_{bus} elements (Ω^{-1}).
- $y_{ss,j}$ Feeder segment admittance from HV/MV substation to bus j represented by Y_{bus} elements (Ω^{-1}).
- pf Unity system power factor.
- $r_{f_{ij}}$ Branch repair duration (hour).
- $\lambda_{f_{ij}}$ Branch failure rate.
- $I_{f_{ij}}$ Branch Current (A).
- $U_{f_{ij}}$ Branch unavailability (hour).
- $V_{i,t}$ Bus Voltage at year t (KV).
- V_n Network rated voltage (KV).
- $\Delta V_{ij,t}$ Voltage drop from bus i to j (KV).
- ΔV_{\max} Maximum permissible voltage drop (KV).
- t A typical cost function in terms of year t .
- $InfR$ The inflation rate.
- $IntR$ The interest rate.
- t_h Horizon planning period (year).