A comprehensive AC load flow based framework for concurrent planning of transmission networks expansion and reactive power resources: an MILP formulation

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Abstract

Transmission network expansion planning is a challenging problem with the aim to determine the type, location, and time of the equipment to be added to the existing transmission network in order to desirably meet the increasing load demand. On the other hand, reactive power sources are the other important components of the power system, which are used improve voltage profile and maintain the system stability. This paper, presents a comprehensive AC load flow based framework for concurrent expansion planning of transmission networks and reactive power resources. Although major advances have been made in optimization techniques, finding an optimal solution to a problem of this nature is still challenging. Using some linearization techniques, a mixed-integer linear programming (MILP) model for transmission expansion and reactive power planning is proposed. In order to show the influence of the concurrent transmission network expansion and reactive power resources planning in reducing investment costs, simulation studies and analysis of the numerical results are carried out on the Garver 6-Bus and the standard IEEE 24-bus test system.

Keywords: Transmission network expansion planning; reactive power resource; linearization; N-1 contingency criteria.

1 Nomenclature

i, j	Index of bus
k	Index of line
L	Index of corridor
ST	$L-1$ scanning matrix
P_{Di}	Power demand at bus i
P_{Gi}	Conventional power generation at bus i
$a_{L,k}$	Binary variable which takes the value 1 if the k^{th} circuit is installed in transmission corridor L and 0 otherwise
$n_L^{\max}$	Maximum number of lines allowed in each corridor
$n_L^{\min}$	Minimum number of lines allowed in each corridor
$n_L^{\max}$	Maximum number of lines allowed in each corridor
$n_L^{\min}$	Minimum number of lines allowed in each corridor
Ω_L	Set of all transmission corridors L
$P_{L,k,s}^{from}$, $Q_{L,k,s}^{from}$, $S_{L,s}^{from}$	Active, reactive and apparent power flow in the k^{th} circuit of transmission corridor L at bus i under contingency s
$P_{L,k,s}^{to}$, $Q_{L,k,s}^{to}$, $S_{L,s}^{to}$	Active, reactive and apparent power flow in the k^{th} circuit of transmission corridor L at bus j under contingency s
g_L	Conductance of line, a positive value.
b_L	Series admittance of line , a negative value.
bc_L	Shunt admittance of line , a positive value.
$V_{i,s}$	Bus voltage magnitude in p.u. at bus
$\theta_{i,s}$	Bus voltage angle at bus i
$\theta_{j,s}$	Bus voltage angle at bus j
$V^{\max}$	Upper bound on the voltage magnitude
$V^{\min}$	Lower bound on the voltage magnitude
$\theta^{\max}$	Maximum Bus voltage angle at bus i
$\theta^{\min}$	Minimum Bus voltage angle at bus i
$S_L^{\max}$	MVA rating each lines of corridor L
$P_{Gi}{}^{\max}$	Maximum active power output of generator at bus i
$P_{Gi}{}^{\min}$	Minimum active power output of generator at bus i
$Q_{Gi}{}^{\max}$	Maximum reactive power output of generator at bus i
$Q_{Gi}{}^{\min}$	Minimum reactive power output of generator at bus i
$PC_{Di,s}$	Active load shedding at bus i
$QC_{Ci,s}$	Reactive load shedding at bus i
RS_{Ci}	Maximum allowable values for installation of these sources in bus i
$QC_{Ci}^{\max}$	Maximum reactive load shedding at bus i in all contingency

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$PC_{Di}^{\max}$	Maximum active load shedding at bus i in all contingency
c_L	Cost of each circuits at corridor L
γ_P	Penalty for active load shedding
γ_Q	Penalty for reactive load shedding
ΔV_i	Voltage magnitude deviation from 1 p.u. at bus
$\Delta V_i^{\max}$	Upper bound on the voltage magnitude deviation.
$\Delta V_i^{\min}$	Lower bound on the voltage magnitude deviation
M	Disjunctive factor, a large positive number.
$Pl_{L,k,s}$	Active power loss on line k at corridor L under contingency s
$Ql_{L,k,s}$	reactive power loss on line k at corridor L under contingency s
$W(d)$	Slope of the d^{th} piecewise linear block.
$\Delta\theta_L(d)$	d^{th} Linear block of angle difference across lines at corridor L
θ_L^+, θ_L^-	Nonnegative slack variables used to replace $\theta_i - \theta_j$
d	Index of blocks used for piecewise linearization
$\theta^{\max}$	Maximum angle difference across a line

2 Introduction

One of the principal components of power system planning is the transmission expansion planning (TEP), which is intended to determine the type, time, and place of constructing new lines and substations. The planning is done according to the load, generation and network conditions, so that, while fulfilling the technical constraints and operational requirements and achieving an acceptable level of reliability, the network expansion and utilization can be obtained at minimum cost [1, 2].

In [3, 4] classification of transmission system expansion planning models is presented. Besides, in [5] challenges on transmission planning modeling and solution approaches are reviewed. Reference [6] presents a multi-operator evolutionary algorithms, as a metaheuristic approach, to solve transmission network expansion planning. In [7] the possibility of applying AC-based models on the TEP problem has been analyzed. In [8]a non-deterministic framework to cope with the uncertain nature of capital costs and electricity demands using the information-gap decision theory is presented. In [1] an AC transmission network expansion planning method is proposed. The AC power flow formulation is based on the semi-definite programming relaxation. A mixed-integer linear programming (MILP) for TEP problem is proposed in [9]. Also, in [10] a linearization method is presented to transform the nonlinear AC model of transmission network expansion planning to the easy to solve linear one.

In recent years, improvement of power system voltage conditions have been repeatedly addressed in literature. It is well established that provided that the bus voltage is not placed in the secure range, the usage of reactive power sources, such as capacitors and switchable reactors, synchronous condensers, and static reactive power compensators, such as controlling equipment is necessitated. These actions, could be a helpful step to maintain the voltage within the acceptable limits [11]. The reactive power planning (RPP) studies are part of power system studies, which follow the network expansion planning to determine the optimal location and capacity of the reactive power sources in order to improve the systems' voltage profile [12].

Investigations executed in the field of power network planning have rarely studied the simultaneous transmission network expansion and reactive power planning. Moreover, metaheuristic algorithms have mostly been employed in solving these problems [13-15].

In [13] the AC model of power system is considered in the combined transmission expansion planning and reactive power planning problem in a deregulated market. Particle swarm optimization (PSO) method is used to solve the proposed planning problem which is a nonlinear mixed integer optimization problem. Also, reference [15] develops a hybrid heuristic method to solve concurrent transmission network expansion and reactive power planning. The main reason for using metaheuristic algorithms is that simultaneous transmission network expansion and reactive power planning is a mixed integer nonlinear problem and its solving process by using mathematical methods will come with many difficulties.

Nevertheless, mathematical programming approaches for solving linear problems will act strongly and a global optimum solution will be guaranteed and this is opposed to metaheuristic methods that may be trapped in a local optimum.

In terms of security constraints, the NERC criterion indicates that the power grid should be operational in case of outage an equipment from the system. In fact, the *N*-1 contingency criterion should not be violated. The *N*-1 criterion has been used for measuring the level of security in expansion plan of many papers in the field of expansion planning [16-18]. In this paper, the *N*-1 criterion using the binary matrices is used.

Given the issues discussed, contribution of this paper is offering a static linear model for simultaneous transmission network expansion and reactive power planning by using the AC load flow model.

This paper, completes and extends the proposed model of [8, 9] and proposes a linearized model for static concurrent transmission network expansion and reactive power planning (TEP&RPP). In brief, the main contributions of this paper with respect to the previously published works in the area can be summarized as follows.

- Proposing an AC load flow based framework for simultaneous transmission network expansion and reactive power planning;
- Linearizing the nonlinear parts of the model and proposing an MILP model.

The rest of the paper has been structured as follows. In Section 2, modeling of the *N*-1 criterion in transmission network expansion planning is presented. a nonlinear model for the simultaneous transmission network expansion and reactive power planning is proposed in Section 3. Using a series of linearization techniques for linearizing the model is discussed in Section 4. Numerical studies on two different test systems are presented in Section 5 and finally, conclusions are presented in Section 6.

2 Modeling the N-1 criterion

Security constraints in the steady state of transmission expansion planning are often recognized by the *N*-1 criterion. The *N*-1 criterion indicates that the transmission system should be resistant to the exclusion of every single equipment, and all constraints of the planning problem should be within their allowed ranges. In a network with L transmission lines, L+1 different modes exist. By defining the zero-one ST matrix, these modes will be modeled as follow [19]:

$$ST=\left[\begin{array}{c|ccc}1 & 0 & & 1\\ \vdots & & \ddots & \\ 1 & 1 & & 0\end{array}\right]_{L\times L+1} \tag{1}$$

In this matrix, zero valued elements mean that the intended transmission line is out of service, and the elements valued with “1”, reflect the normal state of respective line. Each of ST matrix columns represents a specific state of the network. In the first column of the matrix, the values of all elements are 1. This column shows the basic state of network in which all transmission lines are in service.

3 Nonlinear formulation

In this section, the mathematical formulation of proposed concurrent transmission network expansion and reactive power planning is presented. The proposed formulation is based on the AC load flow model.

Consider a transmission network with n buses and load demand P_{Di}, dispatchable generation P_{Gi}. Transmission lines are defined along the specified transmission corridor. Each transmission corridor between bus i and j has a minimum and maximum number of allowed circuits. The minimum number refers to the existing circuits of the transmission corridor $i-j$ of the main network. If there would be no circuit in a particular corridor, this value will be set to zero. Also, $a_{L,k}$ is a binary variable. If the k^{th} circuit is connected in the L^{th} corridor, this variable will take the value of 1; otherwise, it will take zero, as shown below [20].

$$a_{L,k}\in\{0,1\},\quad L\in\Omega_L,\ k=1,\dots,n_L^{\max} \tag{2}$$

$$n_L^{\min}\le\sum_{k=1}^{n_L^{\max}} a_{L,k}\le n_L^{\max}\qquad L\in\Omega_L,\ k=1,\dots,n_L^{\max} \tag{3}$$

Sequential connection of circuits in each transmission corridor will be adjusted through the following logic constraint.

$$a_{L,k}\le a_{L,(k-1)},\quad L\in\Omega_L,\ k=2,\dots,n_L^{\max} \tag{4}$$

The current active and reactive power in the k^{th} transmission line in the L^{th} corridor, which will be determined between bus i and j, can be calculated through the AC load flow equations as follows. In the equations, index s represents various states or occurrence of various contingencies of the existing line in the network.

$$\begin{aligned}P_{L,k,s}^{from}&=(V_{i,s}^2 g_L-V_{i,s}V_{j,s}(g_L\cos(\theta_{i,s}-\theta_{j,s})\\&+b_L\sin(\theta_{i,s}-\theta_{j,s})))a_{L,k}.ST_{L,s}\end{aligned} \tag{5}$$

$$\begin{aligned}Q_{L,k,s}^{from}&=(-V_{i,s}^2(b_L+bc_L)+V_{i,s}V_{j,s}\times\\&(b_L\cos(\theta_{i,s}-\theta_{j,s})-g_L\sin(\theta_{i,s}-\theta_{j,s})))a_{L,k}.ST_{L,s}\end{aligned} \tag{6}$$

$$\begin{aligned}P_{L,k,s}^{to}&=(V_{j,s}^2 g_L-V_{i,s}V_{j,s}(g_L\cos(\theta_{i,s}-\theta_{j,s})\\&-b_L\sin(\theta_{i,s}-\theta_{j,s})))a_{L,k}.ST_{L,k}\end{aligned} \tag{7}$$

$$\begin{aligned}Q_{L,k,s}^{to}&=(-V_{j,s}^2(b_L+bc_L)+V_{i,s}V_{j,s}\times\\&(b_L\cos(\theta_{i,s}-\theta_{j,s})+g_L\sin(\theta_{i,s}-\theta_{j,s})))a_{L,k}.ST_{L,s}\end{aligned} \tag{8}$$

Note that, if the k^{th} circuit has not been established, the current active and reactive power in the lines will be adjusted to zero $(a_{L,k}=0)$; and if the k^{th} circuit has been established $(a_{L,k}=1)$, the current power in the lines will be calculated by using the AC load flow equations.

The constraints related to the voltage magnitude and the angle of bus voltages are according to equations (9) and (10), respectively.

$$V^{\min}\le V_{i,s}\le V^{\max} \tag{9}$$

$$\theta^{\min}\le\theta_{i,s}\le\theta^{\max} \tag{10}$$

Calculating the apparent power of the lines as well as the related constraints, by calculating the current power in the lines, through equations (5) to (8), can be done as follow.

$$S_{L,s}^{from}=\sqrt{(P_{L,s}^{from})^2+(Q_{L,s}^{from})^2} \tag{11}$$

$$S_{L,}^{to}=\sqrt{(P_{L,s}^{to})^2+(Q_{L,s}^{to})^2} \tag{12}$$

$$S_{L,s}^{from}\le S_L^{\max} \tag{13}$$

$$S_{L,s}^{to}\le S_L^{\max} \tag{14}$$

The generating power of plants are considered according to equations (15) and (16) and within their corresponding ranges.

$$P_{Gi}^{\min} \le P_{Gi,s} \le P_{Gi}^{\max} \quad ,\forall i \tag{15}$$

$$Q_{Gi}^{\min} \le Q_{Gi,s} \le Q_{Gi}^{\max} \quad ,\forall i \tag{16}$$

The active and reactive power balance equations are as follow.

$$P_{Gi,s} - (P_{Di} - PC_{Di,s}) = \sum_{L \in i} \sum_{k} \left(P_{L,k,s}^{from} + P_{L,k,s}^{to} \right) \tag{17}$$

$$Q_{Gi,s} - (Q_{Di} - QC_{Ci,s}) = \sum_{L \in i} \sum_{k} \left(Q_{L,k,s}^{from} + Q_{L,k,s}^{to} \right) \tag{18}$$

Where $PC_{Di,s}$ and $QC_{Ci,s}$ refer to the active and reactive load shedding at bus i. These variables are limited to their corresponding available quantities.

$$0 \le PC_{Di,s} \le P_{Di} \quad ,\forall i,s \tag{19}$$

$$0 \le QC_{Ci,s} \le RS_{Ci} \quad ,\forall i,s \tag{20}$$

$$QC_{Ci}^{\max} \ge QC_{Ci,s} \quad ,\forall s \tag{21}$$

$$PC_{Di}^{\max} \ge PC_{Di,s} \quad ,\forall s \tag{22}$$

In equations (15) and (16), the minimum and maximum range of the buses that conventional generators are not connected to them, is set to zero. Similarly, for the buses that have no connection to the load, the right side of equation (19) should be set to zero. In this formulation, reactive sources are modeled with the reactive load shedding. Through this method, the number of variables to be added to the problem will be decreased and consequently, the computational load will be lowered. In this study, reactive sources have been modeled by using continuous variables. Despite this fact, these sources are discrete in nature, which is, of course, an acceptable assumption to simplify the problem. In equation (20), the values of reactive sources are limited to the maximum allowable values for installation of these sources in each bus [21]. In equation (21) and (22) the maximum active and reactive load shedding among all possible outage will be determined.

The total constraints of the simultaneous transmission network expansion and reactive power planning include the constraints (2) to (22). The objective function in this nonlinear model will be as follows.

$$\min \quad \sum_{L} \sum_{k=n_L^{min}+1}^{n_L^{max}} c_L a_{L,k} + \sum_{i=1}^{n} \left(\gamma_P PC_{Di}^{max} + \gamma_Q QC_i^{max} \right) \tag{23}$$

The first expression relates to the investment cost of the lines. Index k is started from $n_L^{\min}+1$, because the construction cost of the current lines is obviously zero. The second expression relates to the active and reactive load shedding penalty. The value of γ_P, compared to the other costs, will be chosen as a great value, so that the active load shedding reaches zero. But γ_Q will be considered equal to the cost of the reactive power sources. Thus, the reactive power load shedding amount occurring in a particular bus is the same as the amount of installed reactive power sources.

4 Linearization of the model

The approximation obtained from traditional DC model will significantly simplify the AC model. However, in some cases, it will decrease the model accuracy. In order to improve the accuracy and validity of the model, and also reduce the computational costs, a linear representation of the AC load flow model and network losses is presented in this section.

4.1 Linearization of the AC load flow equations

Linearization of the equations of the power lines will be carried out based on Taylor series and by considering the following valid assumptions [9].

1) Bus voltage magnitudes are always close to 1 p.u.

2) Since the angle difference along the line is small, therefore, it can be considered that $\sin\left(\theta_i - \theta_j\right) \approx \theta_i - \theta_j$ and $\cos\left(\theta_i - \theta_j\right) \approx 1$. This assumption at the transmission level, where the active power flow is the dominant part of the apparent power flow, is quite valid.

According to the above assumptions, the voltage magnitude of the buses can be considered as follows.

$$V_i = 1 + \Delta V_i \tag{24}$$

where ΔV is expected to allocate small values to itself from the below interval.

$$\Delta V_i^{\min} \le \Delta V_i \le \Delta V_i^{\max} \tag{25}$$

By substituting equation (24) in equations (5) and (6), and ignoring the expressions with higher degrees, the following equations are achieved

$$\begin{aligned} &P(\Delta V_i, (\theta_i - \theta_j)) \approx (1 + 2\Delta V_i) g_L \\ &-(1 + \Delta V_i + \Delta V_j)(g_L + b_L(\theta_i - \theta_j)) \end{aligned} \tag{26}$$

$$\begin{aligned} &Q(\Delta V_i, (\theta_i - \theta_j)) \approx -(1 + 2\Delta V_i)(b_L + bc_L) + \\ &(1 + \Delta V_i + \Delta V_j)(b_L - g_L(\theta_i - \theta_j)) \end{aligned} \tag{27}$$

It is noted that equations (26) and (27) are still nonlinear. As it is expected that the values of ΔV_i, ΔV_j and $(\theta_i - \theta_j)$ would be small, the expressions such as $\Delta V_i(\theta_i - \theta_j)$ and $\Delta V_j(\theta_i - \theta_j)$ can be ignored, similar to the second-order expressions. The linearized load flow equations for line k in corridor L, between buses i and j, are as follows.

$$P_{L,k,s} = (\Delta V_{i,s} - \Delta V_{j,s}) g_L - b_L \left(\theta_{i,s} - \theta_{j,s} \right) \tag{28}$$

$$\begin{aligned} Q_{L,k,s} = &-(1 + 2\Delta V_{i,s}) bc_L - (\Delta V_{i,s} - \Delta V_{j,s}) b_L \\ &- g_L \left(\theta_{i,s} - \theta_{j,s} \right) \end{aligned} \tag{29}$$

Hence, the linear equations of the AC load flow will be converted into equations (30) and (31).

$$\begin{aligned} &-M(1 - a_{L,k}.ST_{L,s}) \le P_{L,k,s}^{from} - P(\Delta V_{i,s}, (\theta_{i,s} - \theta_{j,s})) \\ &\le M(1 - a_{L,k}.ST_{L,s}) \end{aligned} \tag{30}$$

$$\begin{aligned} &-M(1 - a_{L,k}.ST_{L,s}) \le Q_{L,k,s}^{from} - Q(\Delta V_{i,s}, (\theta_{i,s} - \theta_{j,s})) \\ &\le M(1 - a_{L,k}.ST_{L,s}) \end{aligned} \tag{31}$$

$$-S_L^{\max} a_{L,k}.ST_{L,s} \le P_{L,k,s}^{from} \le S_L^{\max} a_{L,k}.ST_{L,s} \tag{32}$$

$$-S_L^{\max} a_{L,k}.ST_{L,s} \le Q_{L,k,s}^{from} \le S_L^{\max} a_{L,k}.ST_{L,s} \qquad (33)$$

In equations (32) and (33), the line flow has been limited to the maximum capacity of the line. In equations (5) to (8), multiplication of the binary variable $a_{L,k}$ will lead to the nonlinearity of the model. Thus, in equations (30) and (31), having defined parameter *M*, which in comparison to the other parameters of the problem will have a large value, it will be placed within the equation ranges of variable $a_{L,k}$. The equations, corresponding to $P_{L,k,s}^{to}$ and $Q_{L,k,s}^{to}$, will be calculated similar to equations (30)-(33).

In order to achieve a completely linear model, it is necessary to linearize the nonlinear equations (11) and (12). In the following, consider a simple method for linearizing the equations in the form of $x^2 + y^2 \le R^2$ (a circle with the radius *R* in Cartesian coordinates), as displayed in Fig. 1. Consider a regular convex *m*-sided polygon inscribed in this circle. The equation, which connects point A to B, is as follows [10].

$$y - R\times\sin(2\pi h/m) = \frac{\sin(2\pi h/m) - \sin(2\pi/m(h-1))}{\cos(2\pi/h) - \cos(2\pi/m(h-1))} \times\left(x - R\cos(2\pi h/m)\right) \qquad (34)$$

By a simple change in equation (34), equation (35) will be achieved. By assuming $\alpha = 2\pi h/m$ and $\beta = 2\pi h/m(h-1)$, equation (35) will be simplified as equation (36). The large number of sectors, or in fact, the large number of the convex polygon's sides, will improve the accuracy of the solution, but will lead to more computation time. In this paper, *m = 32* has been used to approximate this circle.

$$\begin{aligned}&(\sin(2\pi h/m) - \sin(2\pi/m(h-1)))x\\ &-(\cos(2\pi h/m) - \cos(2\pi/m(h-1)))y\\ &-R\times\sin(2\pi/m) = 0\end{aligned} \qquad (35)$$

$$\begin{aligned}&\left(\sin\alpha - \sin\beta\right)x - \left(\cos\alpha - \cos\beta\right)y\\ &-R\times\sin(\alpha-\beta) = 0\end{aligned} \qquad (36)$$

Therefore, the linearized equivalent of equations (11)-(14), are the equations (37), (38).

$$\begin{aligned}&(\sin(2\pi h/m) - \sin(2\pi(h-1)/m))P_{L,k,s}^{from}\\ &-(\cos(2\pi h/m) - \cos(2\pi(h-1)/m))Q_{L,k,s}^{from}\\ &-\sin(2\pi/m)S_L^{\max} \le 0\end{aligned} \qquad (37)$$

$$\begin{aligned}&(\sin(2\pi h/m) - \sin(2\pi(h-1)/m))P_{L,k,s}^{to}\\ &-(\cos(2\pi h/m) - \cos(2\pi(h-1)/m))Q_{L,k,s}^{to}\\ &-\sin(2\pi/m)S_L^{\max} \le 0\end{aligned} \qquad (38)$$

4.2 Linearization of line losses

In transmission expansion planning studies, by the assumption that transmission line losses are limited and negligible, usually the DC load flow model is used. This assumption can be problematic in large-scale and wide networks and lead to a situation, where the network expansion plan is different from the actual optimal calculation is hidden, in this linearized model and based on the considered assumptions, it is required to calculate the network losses with the aid of a different method. With regard to the second hypothesis in Section 4.1, and regardless of the higher order expressions, and also by ignoring the lines' capacitance against the inductive reactance, the network losses can be calculated by using equations (39) and (40).

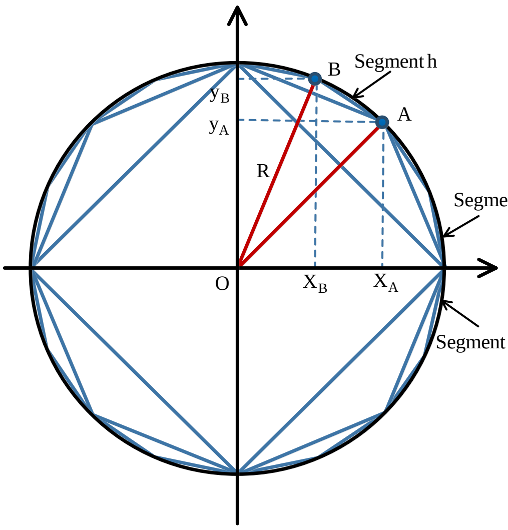

Fig. 1. A circle with the radius *R* [10]

$$PL_{L,k} \approx g_L(\theta_i - \theta_j)^2 \qquad (39)$$

$$QL_{L,k} \approx -b_L(\theta_i - \theta_j)^2 \qquad (40)$$

It should be noted that equations (39) and (40) are nonlinear and nonconvex constraints. In order to linearize these equations, the piecewise linearization technique will be used. As shown in Fig. 2. in this method, the curve $(\theta_i - \theta_j)^2$ has been divided into *D* pieces, and each piece is approximated by a segment line. The linear model of losses will be achieved as follows [9].

$$(\theta_i - \theta_j)^2 \approx \sum_{d=1}^{D} W(d)\Delta\theta_L(d) \;\;, d = 1,\ldots,D \qquad (41)$$

$$\left(\theta_i - \theta_j\right) = \theta_L^+ - \theta_L^- \qquad (42)$$

$$\left|\theta_i - \theta_j\right| = \theta_L^+ + \theta_L^- \qquad (43)$$

$$\left|\theta_i - \theta_j\right| = \sum_{d=1}^{D} \Delta\theta_L(d) \qquad (44)$$

$$0 \le \Delta\theta_L(d) \le \frac{\theta^{\max}}{D} + (1 - a_L^{(k)})\pi \qquad (45)$$

$$W(d) = (2d-1)\theta^{\max}/D \qquad (46)$$

$$\theta_L^+ \ge 0 \;\;, \theta_L^- \ge 0 \qquad (47)$$

where $W(d)$ and $\Delta\theta_L(d)$ are the gradient and size of the d^{th} block of the angle difference of the corridor between bus *i* and *j*, respectively. The slope of each block is calculable using equation (46). In addition, equation (45) limits the size of each block. For the candidate lines, which would not be selected for installation, this constraint should not impose any limitation. Thus, the second expression of the right side of equation (45) has been added to serve this purpose. Also, θ_L^+ and θ_L^- are two positive variables, which have been used to get rid of the absolute value function that causes the problem to be nonlinear.

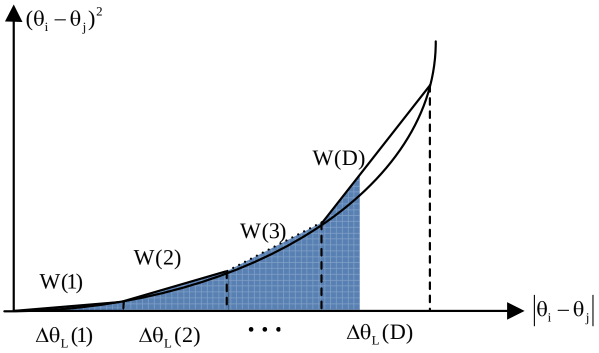

Fig. 2. The piecewise linearization of $(\theta_i - \theta_j)^2$ [9]

Using equations (41)-(47), active and reactive power losses of the lines can be calculated as follow.

$$0 \le -Pl_{L,k,s} + g_L \sum_{d=1}^{D} K_d \Delta\theta_{L,d,s} \le M(1 - a_{L,k}.ST_{L,s}) \tag{48}$$

$$0 \le -Ql_{L,k,s} - b_L \sum_{d=1}^{D} K_d \Delta\theta_{L,d,s} \le M(1 - a_{L,k}.ST_{L,s}) \tag{49}$$

$$0 \le Pl_{L,k,s} \le a_{L,k}.ST_{L,s}.g_L(\theta^{\max})^2 \tag{50}$$

$$0 \le Ql_{L,k,s} \le -a_{L,k}.ST_{L,s}.b_L(\theta^{\max})^2 \tag{51}$$

Equations (48) and (49) are related to the calculation of line losses and in order to prevent the nonlinearity of the model, the big M linearization technique has been used. Similarly, in equations (50) and (51), the line losses will be limited to their corresponding maximum value. By calculating the losses, the active and reactive power balance equations would be as equations (52) and (53). In these equations, losses are split in half, and each half is dedicated to one of the buses that is connected to the related line.

$$P_{Gi,s} - (P_{Di} - PC_{Di,s}) - 0.5\sum\sum_{k} PL_{L,k,s} = \sum_{L\in i}\sum_{k}\left(P_{L,k,s}^{from} + P_{L,k,s}^{to}\right) \tag{52}$$

$$Q_{Gi,s} - \left(Q_{Di} - QC_{i,s}\right) - 0.5\sum_{L\in i}\sum_{k} QL_{L,k,s} = \sum_{L\in i}\sum_{k}\left(Q_{L,k,s}^{from} + Q_{L,k,s}^{to}\right) \tag{53}$$

5 Numerical results

In the previous section, by using a series of linearization techniques, a mixed integer linear programming (MILP) model for the problem of simultaneous transmission network expansion and reactive power planning was obtained. In this part, the performance of the proposed model on the Garver 6-bus test system and the IEEE standard 24-bus test system is tested and the obtained results are analyzed. Numerical studies are carried out using the CPLEX solver in GAMS software environment [22]. All simulations are done on a 64-bit system with 6GB of RAM and the Core i7 CPU. The optimality criterion for stopping the process and obtaining the final results has been considered as $\varepsilon = 10^{-3}$.

5.1 Garver 6-bus test system

The Garver test system is a small-scale transmission network, which has been addressed by many researchers as the basic network in the majority of works executed in the field of network expansion planning. This network was first proposed by Garver in 1970 [23]. The Garver system has 6 buses, 6 existing transmission lines, 5 load centers with a total electrical load of 760 MW, and 3 power plants with a maximum generation of 1140 MW. The planning horizon is considered to be 10 years. The full details of the existing network and candidate lines are available in [24]. The plant has been considered as connected to the bus 6, in isolation from the main network. The basic structure of the system is shown in Fig. 3.

In this network, it is assumed that the maximum number of allowed lines for installation in each corridor is equal to 3. Thus, in total, there are 39 candidate lines. Bus voltage magnitude will be considered in the range of 0.95 to 1.05. The maximum installation value of the reactive power sources at each bus is 1000 MVAr. The installation cost of reactive power planning is considered 100,000 \$/MVAr. The number of linear blocks for linearization of loss equations, and the equations related to the heat range of lines are *D = 20* and *m = 32*, respectively.

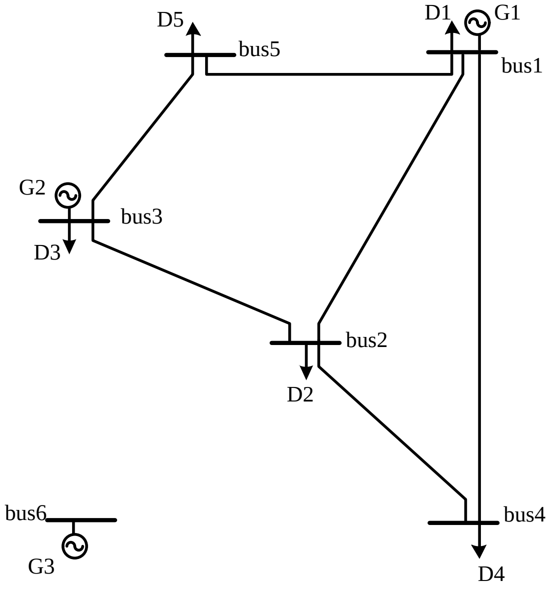

Fig. 3. The Garver 6-bus system

The simulation results of transmission network expansion planning, and the simultaneous transmission network expansion and reactive power planning are shown in Table 1.

According to the results of Table 1, it can be observed that in the network expansion planning method, where the linear model has been employed based on the AC load flow, 6 new transmission lines in the corridors 2-6, 3-5, and 4-6, with a total cost 160M$ have been added to the existing network in order to supply the predicted load. In the simultaneous transmission network expansion and reactive power planning method, 4 lines have been installed in the previous corridors, and also two capacitor compensators in buses 2 and 5 with, respectively, the capacities of 25 MVAr and 9 MVAr, and a total cost of 113.393 M$. The single-line diagram of the network after expanding in both states is shown in Fig. 4. and Fig. 5. The red dotted lines signify the candidate lines, which have been selected for construction. The installed reactive power sources are shown with red squares in the buses.

Table 1. results of TEP and TEP & RPP in the Garver 6-bus test system

Method	Solution		Cost (M$)	Total Cost (M$)
TEP	New lines	$n_{2-6} = 2$ $n_{3-5} = 2$ $n_{4-6} = 2$	160	160
TEP&RPP	New lines	$n_{2-6} = 1$ $n_{3-5} = 1$ $n_{4-6} = 2$	110	113.393
	Installed MVAr	$QC_2 = 25$ $QC_5 = 9$	3.393	

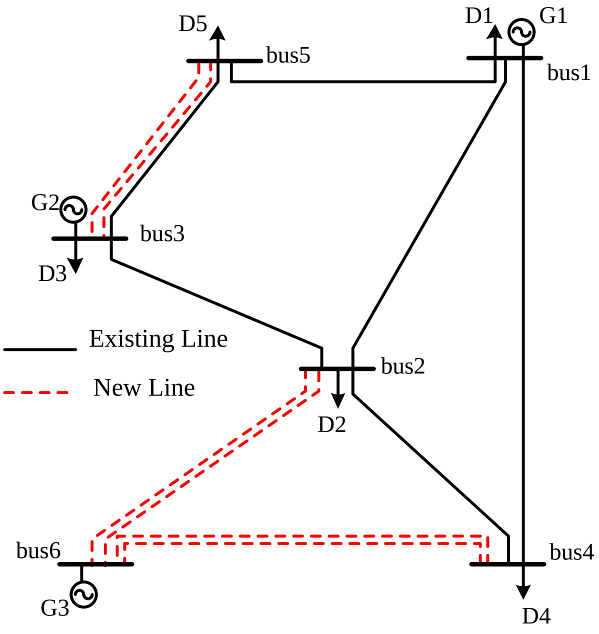

Fig. 4. The Garver 6-bus system after TEP

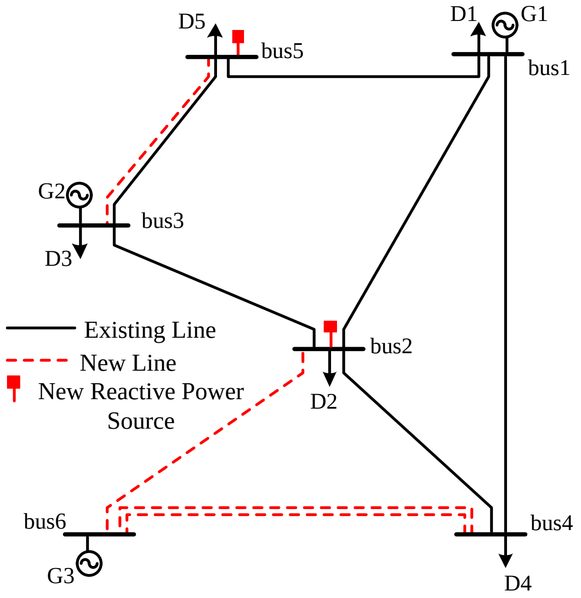

Fig. 5. The Garver 6-bus system after TEP&RPP

5.2 IEEE 24-bus test system

In this system, there are 24 buses, 34 transmission corridors, and 38 transmission lines. 7 candidate corridors have been also considered. The network includes 11 generators and 17 load centers. The consumption and generation values for creating congestion in transmission lines have been increased three times higher with respect to the current network load. The planning horizon is 10 years. The network information as well as the candidate lines have been fully introduced in [24].

The simulation results of transmission network expansion, and the simultaneous transmission network expansion and reactive power planning in the IEEE 24 bus test system (RTS) are shown in Table 2.

Table 2. Results of TEP and TEP&RPP in standard IEEE 24-bus test system

Method	Solution		Cost (M$)	Total Cost (M$)
TEP	New lines	$n_{1-3} = 2$ $n_{2-6} = 2$ $n_{7-8} = 2$ $n_{3-24} = 1$ $n_{6-10} = 2$ $n_{9-12} = 1$ $n_{15-24} = 1$	412	412
TEP&RPP	New lines	$n_{6-10} = 1$ $n_{7-8} = 2$ $n_{9-12} = 1$	98	136.894
	Installed MVAr	$QC_3 = 183.7$ $QC_4 = 35.8$ $QC_9 = 98.6$ $QC_{12} = 70.8$	38.894	

According to the results reported in Table 2, it can be observed that in the transmission network expansion planning method, 11 new transmission lines in corridors 1-3, 2-6, 3-24, 6-10, 7-8, 9-12 and 15-24 with a total cost of 412 M$ have been added to the existing network in order to supply the load. In the simultaneous transmission network expansion and reactive power planning method, 4 new transmission lines in corridors 6-10, 7-8, and 9-12 have been constructed. In addition, the capacitive compensation added to the system is located at buses 3, 4, 9, and 12, with 183.7, 35.8, 98.6, and 70.8 MVAr, respectively. The total cost in this case is equal to 136.894 M$. The single-line diagram of the standard IEEE 24-bus test system after expansion using the two methods has been shown in Fig. 6. and Fig. 7.

5.3 Comparison of the results

In order to show the effectiveness of the proposed model, the obtained results are compared with the results of [21]. As shown in Table 3, in IEEE 24-bus test systems four new transmission lines have been constructed in corridors 6-10, 7-8, and 11-13. In addition, reactive power sources are added to the system at buses 3, 9 and 10, with the capacity of 202, 302, and 27 MVAr, respectively. Also, total cost of planning is 167.1 M$.

Table 3. TEP&RPP results in the IEEE 24-bus test systems in this paper and [21].

	Solution		Cost (M$)	Total Cost (M$)
Purposed method	New lines	$n_{6-10} = 1$ $n_{7-8} = 2$ $n_{9-12} = 1$	98	136.894
	Installed (MVAr)	$QC_3 = 183.7$ $QC_4 = 35.8$ $QC_9 = 98.6$ $QC_{12} = 70.8$	38.89 4	
Method in [21]	New lines	$n_{6-10} = 1$ $n_{7-8} = 2$ $n_{11-13} = 1$	114	167.1
	Installed (MVAr)	$QC_3 = 202$ $QC_9 = 302$ $QC_{10} = 27$	53.1	

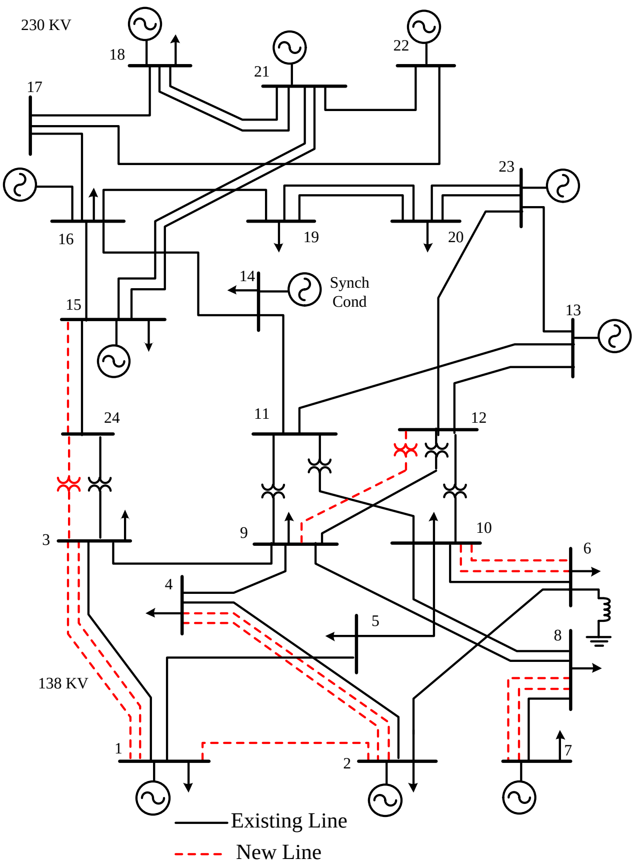

Fig. 6. IEEE 24-bus test system after TEP

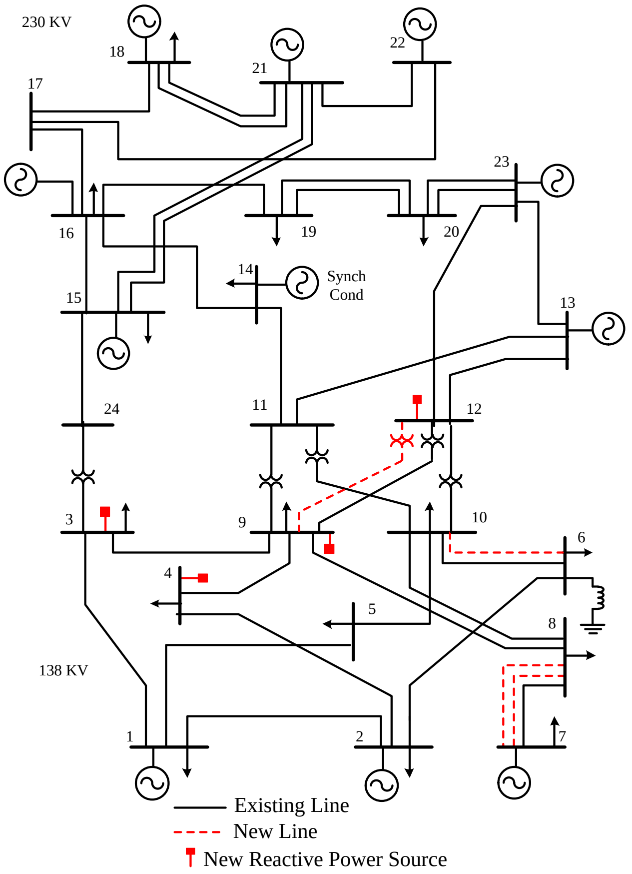

Fig. 7. IEEE 24-bus test system after TEP&RPP

5.4 N-1 security criterion in transmission network expansion planning

In this section, the effect of considering *N*-1 security criterion on the proposed planning model is analyzed. Given that considering the outage of all of system components in the *N*-1 security criterion will greatly increase the computational costs and also because it will lead to a very conservative expansion plan with a very high investment cost, therefore, the most probable and/or sensitive outages will be normally considered in the *N*-1 security criterion.

In this paper, bus voltage magnitude is considered between 0.92 and 1.06 of the nominal value and thermal capacity of lines is considered to be 1.1 times of normal state of the network [9]. In the Garver 6-bus test system, the outage of lines 2-3, 2-6, and 3-5, and in the standard IEEE 24-bus test system, the outage of lines 1-5, 3-24, 11-13, and 12-23 have been considered. The results of considering the *N*-1 criterion in both test systems are shown in Table 4. According to the results, by considering *N*-1 criterion, more transmission lines will be selected for installation and as a result the investment cost will be increased. However, the network will be more secure against the equipment outages. In this case, the planner must do a tradeoff between the investment cost and network security.

Table 4. TEP&RPP results for the Garver 6-bus and IEEE 24-bus test systems with *N*-1 criterion

Test Systems	Solution		Cost (M$)	Total Cost (M$)
Garver 6-bus	New lines	$n_{1-3} = 1$ $n_{1-5} = 1$ $n_{2-3} = 1$ $n_{2-6} = 1$ $n_{3-5} = 1$ $n_{4-6} = 2$	188	199.536
	Installed (MVAr)	$QC_2 = 32.4$ $QC_4 = 83$	11.536	
IEEE 24-bus	New lines	$n_{3-9} = 1$ $n_{6-10} = 2$ $n_{7-8} = 1$ $n_{14-16} = 1$	133	180.059
	Installed (MVAr)	$Qc_3 = 259$ $Qc_4 = 3.6$ $Qc_9 = 207.9$	47.059	

6 Conclusions

This paper presents a mathematical MILP model for simultaneous transmission network expansion and reactive power planning that guarantees the optimality of solution. In the proposed formulation, in order to reduce the complexity of the model and consequently reduce computational burden, reactive power sources are determined according to the required amount of reactive load shedding in each bus of the system. Simulation results on the Garver 6-bus and standard IEEE 24-bus test systems show that simultaneous planning of transmission expansion and reactive power sources, will significantly reduce the investment costs through releasing the transmission lines' capacity.

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