

# Robust finite time stabilization for uncertain switched delay systems with average dwell time

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**Abstract**— This paper investigates the robust finite time stability and finite time stabilization for a class of uncertain switched systems which have time delay. The emphasis of the paper is on the cases where uncertainties are time varying and unknown but norm bounded. By using the average dwell time approach and multiple Lyapunov like functions, delay dependent sufficient conditions for finite time stability of uncertain switched systems with time delay in terms of a set of the linear matrix inequalities are presented. Then, the corresponding conditions are obtained for finite time stabilization of uncertain switched time delay systems via a state feedback controller. The controller is designed by virtue of the linear matrix inequalities and the cone complement linearization method. We solved the problem of uncertainty in uncertain switched time delay systems by resorting to Yakubovich lemma. Finally, numerical examples are provided to verify the effectiveness of the proposed theorem.

**Index Terms**—: Uncertain switched time delay systems; Multiple Lyapunov-like functions; Finite time stabilization; Average dwell time; Cone complement linearization method.

## I. INTRODUCTION

Most of the existing researches related to stability and stabilization of systems have studied Lyapunov asymptotic stability analysis, which is defined over an infinite time interval [1-10]. There are some cases that are concerned about the dynamical behavior of a system over a finite interval of time, such as network congestion control [11], network control systems [12-13] and switched systems [14]. It should be noted that finite-time stability and Lyapunov asymptotic stability are different concepts and they are independent of each other: a system may be finite time stable but not Lyapunov asymptotic stable and vice versa. A system is finite-time stable (FTS) if the system states retain certain prescribed bounds in the fixed time interval under bounded initial conditions [15-16]. Switched systems are a class of hybrid systems which consist of a finite number of subsystems and a switching signal controlling at any time instant that subsystem is active [17]. In the last decades, switched systems have received extensive attention for their practical applications and importance in

theory development [18] such as power electronics [19], network communication [20] and chemical processing [21]. In the mentioned applications, delay plays an important role in the switched systems and therefore it is not avoidable in control design.

Time delays exist in many physical processes which may degrade system performance, cause oscillation and even instability [22]. For switched systems, due to the interaction among continuous dynamics, discrete switching and time delay, the problem of switched time-delay systems (STDS) is more complex than switched systems without time-delay and time delay systems that are without switching [23]. So far, Lyapunov asymptotic stability analysis for STDS [24-28] and finite time stability for non-switched systems [12-13, 15-16, 29-32] have been investigated by many researchers.

For stability analysis of STDS under arbitrary switching, usually the common Lyapunov function (CLF) is used, but this approach is conservative and it is often difficult to find the CLF for all subsystems. The multiple Lyapunov functions (MLF) and the average dwell time (ADT) approach have been suggested as effective tools for reducing conservatism in stability STDS. In order to analyze and synthesis the problem of FTS, often the multiple Lyapunov-like functions are employed. The advantages of multiple Lyapunov-like functions are in their flexibility, because different Lyapunov-like functions can be constructed for various subsystems.

Parameter uncertainty is often met in various practical and technical systems that make it difficult to extend an accurate mathematical model. It has been shown that uncertainty is the source of instability and often causes undesirable performance of control systems [28]. Therefore, robust finite time stability of uncertain switched systems is important in theory and application.

However, compared with numerous researches on Lyapunov stability of STDS [24-28], few results on finite-time stability of STDS [33-39] have been studied in the previous literature. In [33], finite-time stability, finite time boundedness and finite time weighted  $L_2$ -gain for a class of switched systems with sector-bounded nonlinearity and constant time delay have been investigated. The finite-time  $H_\infty$  control problem for a class of discrete-time switched nonlinear systems with time-delay is discussed based on the average dwell time approach in [34]. The problem of finite time control of linear stochastic switched systems with constant time delay are presented in [35]. The

authors of [36] investigated finite-time control for a class of switched delay systems via dynamic output feedback. Finite-time stabilization (FTSz) under asynchronous switching is dealt with a class of switched time-delay systems with nonlinear disturbances via the differential mean value theorem in [37]. In [38], the problem of finite time boundedness for switched linear systems with time-varying delay and external disturbance was discussed based on the Jensen inequality approach and the average dwell time method. Sufficient conditions were obtained in [39] for finite-time filtering of switched linear systems with a mode-dependent ADT by introducing a newly augmented Lyapunov-Krasovskii and considering the relationship between time-varying delays and their upper delay bounds. Up to now, to the best of our knowledge, the issue of finite time stability and finite time stabilization for STDS with uncertainties has received little attention in the previous research. However, in practical engineering, switched systems are commonly subjected to time delay and uncertainties. Moreover, some practical systems are just required that their state trajectories are bounded over a fixed interval. Considering the wide application of switched time delay systems with uncertainties and the requirements for transient behaviors in engineering fields, it is a significant task to investigate finite time stability and stabilization for switched systems with time delay and uncertainties.

In this paper, we consider the problems of FTS and FTSz of linear switched system with time delay and uncertainty. Based on the multiple Lyapunov-like function and average dwell time, sufficient conditions are proposed to guarantee FTS and FTSz. The state feedback controller design problem is solved by using the cone complement linearization (CCL) algorithm.

The remainder of the paper is organized as follows. In section 2, Problem formulation, Definitions and some necessary lemmas are given. In section 3, based on the average dwell time method and multiple Lyapunov-like functions, some new delay-dependent conditions guaranteeing finite-time stability and stabilization of the uncertain switched time-delay system (USTDS) are developed. In Section 4, numerical examples are given to show the validity of the obtained results. Concluding remarks are given in Section 5.

Notations: The notations used in this paper are standard. The symbol ‘\*’ denotes the elements below the main diagonal of a symmetric matrix. The superscript ‘ $T$ ’ stands for matrix transposition.  $R^n$  indicates the  $n$ -dimensional Euclidean space.  $I$  and  $O$  signify the identity matrix and a zero matrix. The notation  $X > 0$  means that  $X$  is real symmetric and positive definite.  $diag\{...\}$  denotes for a block-diagonal matrix. The notation ‘sup’ means the supremum.  $\lambda_{min}(P)$  and  $\lambda_{max}(P)$  stand the minimum and maximum eigenvalues of matrix  $P$ , respectively.

## 2. Preliminaries and Problem formulation

Consider the USTDS as follows

$$\dot{x}(t) = \hat{A}_{\sigma(t)}x(t) + \hat{A}_{d\sigma(t)}x(t-d) + \hat{B}_{\sigma(t)}u(t)x(t) = \varphi(t), \quad t \in [-d, 0] \quad (1)$$

Where  $x(t) \in R^n$  is the state vector,  $u(t) \in R^m$  is the control input,  $\varphi(t)$  is a continuous vector-valued initial function on  $[-d, 0]$ ,  $d > 0$  is the constant time delay.  $\sigma(t): [0, \infty) \rightarrow$

$L = \{1, 2, \dots, N\}$  is a switching signal which is right continuous and piecewise constant and  $N$  is the number of subsystems.

Corresponding to the switching signal  $\sigma(t)$ , the switching sequence as follows

$$\{(i_0, t_0), (i_1, t_1), \dots, (i_k, t_k), \dots, | i_k \in L, k = 0, 1, \dots\}$$

Where  $t_0$  is the initial time, the  $i_k$ th subsystem is activated when  $t \in [t_k, t_{k+1})$ .

For each  $i \in L$ ,  $\hat{A}_i, \hat{A}_{di}, \hat{B}_i$  are uncertain real-valued matrices with appropriate dimensions. We assume that the uncertainties are norm bounded as follows

$$\hat{A}_i = A_i + \Delta A_i, \quad \hat{A}_{di} = A_{di} + \Delta A_{di}, \quad \hat{B}_i = B_i + \Delta B_i \quad (2)$$

$$[\Delta A_i \quad \Delta A_{di} \quad \Delta B_i] = M_i F_i(t) [N_i \quad N_{di} \quad N_{Bi}] \quad (3)$$

Where  $A_i, A_{di}, B_i, M_i, N_i, N_{di}$  and  $N_{Bi}$  are known real-valued constant matrices with appropriate dimensions.  $F_i(t)$  is unknown and possibly time varying matrix satisfying

$$F_i^T(t)F_i(t) \leq I \quad (4)$$

Definition 1 [40]. For any  $T \geq t \geq 0$ , let  $N_\sigma(t, T)$  indicate the switching number of  $\sigma(t)$  over  $(t, T)$ . If  $N_\sigma(t, T) \leq N_0 + \frac{T-t}{\tau_a}$  holds for  $\tau_a > 0$  and an integer  $N_0 \geq 0$ , then  $\tau_a$  is called an average dwell time and  $N_0$  is called the chattering bound. For the sake of convenience and following the common practice in the literature, we consider  $N_0 = 0$ .

Definition 2 [41]. Switched system (1) with  $u(t) = 0$  is said to be finite time stable with respect to  $(c_1, c_2, T_f, R, \sigma(t))$ , where  $0 \leq c_1 < c_2$ ,  $T_f$  is a time constant,  $R$  is a positive definite matrix and  $\sigma(t)$  is a switching signal, if

$$\sup_{-d \leq \theta \leq 0} \{x^T(\theta)Rx(\theta)\} \leq c_1 \Rightarrow x^T(t)Rx(t) < c_2 \quad \forall t \in (0, T_f] \quad (5)$$

Remark 1. Switched system (1) with  $u(t) = 0$  is said to be uniformly finite time stable with respect to  $(c_1, c_2, T_f, R)$ , if condition (5) holds for any switching signal. The meaning of ‘uniformly’ is with respect to the switching signal, rather than the time [42].

Lemma 1 (Schur complement [43]). Let  $G, S$  and  $R$  be given matrices such that  $R > 0$ . Then

$$\begin{bmatrix} G(x) & S(x) \\ S^T(x) & -R(x) \end{bmatrix} < 0 \Leftrightarrow S(x)R^{-1}(x)S^T(x) + G(x) < 0 \quad (6)$$

Lemma 2 (Yakubovich Lemma [44]). Let  $\pi_0(x)$  and  $\pi_1(x)$  be two quadratic matrix functions on  $R^n$ , and  $\pi_1(x) \leq 0$  for all  $x(t) \in R^n - \{0\}$ . Then  $\pi_0(x) < 0$  holds for all  $x(t) \in R^n - \{0\}$  if and only if there exist the constant  $\varepsilon \geq 0$  such that

$$\pi_0(x) - \varepsilon\pi_1(x) < 0, \quad \forall x(t) \in R^n - \{0\} \quad (7)$$

## 3. Main results

In this section, the problem of finite time stability for STDS with uncertainties is investigated and then robust finite time stabilization analysis of the USTDS via state feedback is studied.

3. Finite time stability analysis

Consider the USTDS without the control input. In this subsection, sufficient conditions which guarantee finite time stability of system (1) are given.

Theorem 1. Consider uncertain switched system (1) with  $u(t) = 0$ . If for each  $i \in L$ , there exist positive definite symmetric matrices  $P_i, Q_i$  with appropriate dimensions and positive scalars  $\alpha, \lambda_1, \lambda_2, \lambda_3, \mu \geq 1$  such that

$$\begin{bmatrix} \varphi_{i,11} & P_i A_{di} & N_i^T & 0 & P_i M_i & P_i M_i \\ * & -e^{\alpha d} Q_i & 0 & N_{di}^T & 0 & 0 \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \quad (8)$$

$$(\lambda_2 + de^{\alpha d} \lambda_3) c_1 < \lambda_1 c_2 e^{-\alpha T_f} \quad (9)$$

$$P_i \leq \mu P_j, \quad Q_i \leq \mu Q_j, \quad \forall i, j \in I \quad (10)$$

Then, under the following average dwell time

$$\tau_a > \tau_a^* = \frac{T_f L n \mu}{\ln(\lambda_1 c_2 e^{-\alpha T_f}) - \ln[(\lambda_2 + de^{\alpha d} \lambda_3) c_1]} \quad (11)$$

The USTDS is finite time stable with respect to  $(c_1, c_2, T_f, R, \sigma(t))$ , where

$$\varphi_{i,11} = A_i^T P_i + P_i A_i - \alpha P_i + Q_i$$

$$\lambda_1 = \min_{\forall i \in L} (\lambda_{\min}(\tilde{P}_i)) = \min_{\forall i \in L} (\lambda_{\min}(R^{-\frac{1}{2}} P_i R^{-\frac{1}{2}}))$$

$$\lambda_2 = \max_{\forall i \in L} (\lambda_{\max}(\tilde{P}_i)) = \max_{\forall i \in L} (\lambda_{\max}(R^{-\frac{1}{2}} P_i R^{-\frac{1}{2}}))$$

$$\lambda_3 = \max_{\forall i \in L} (\lambda_{\max}(\tilde{Q}_i)) = \max_{\forall i \in L} (\lambda_{\max}(R^{-\frac{1}{2}} Q_i R^{-\frac{1}{2}}))$$

Proof. Choose a Lyapunov-like function as follows

$$V(t) = V_{\sigma(t)}(t)$$

The form of each  $V_i(x)(\forall i \in L)$  is given by

$$V_i(t) = V_{1i}(t) + V_{2i}(t) \quad (12)$$

Where

$$V_{1i}(t) = x^T(t) P_i x(t), \quad V_{2i}(t) = \int_{t-d}^t x^T(s) e^{\alpha(t-s)} Q_i x(s) ds \quad (13)$$

Taking the derivative of  $V(t)$  with respect to  $t$  along the trajectory of the unforced switched system (1) yields

$$\begin{aligned} \dot{V}_{1i}(t) = & x^T(t) (A_i^T P_i + P_i A_i) x(t) + \\ & x^T(t) (M_i F_i(t) N_i)^T P_i x(t) + x^T(t) P_i (M_i F_i(t) N_i) x(t) \\ & + x^T(t-d) A_{di}^T P_i x(t) + x^T(t) P_i A_{di} x(t-d) + x^T(t-d) \\ & (M_i F_i(t) N_{di})^T P_i x(t) + x^T(t) P_i (M_i F_i(t) N_{di}) x(t-d) \end{aligned} \quad (14)$$

$$\dot{V}_{2i}(t) = \alpha V_{2i}(t) + x^T(t) Q_i x(t) - x^T(t-d) e^{\alpha d} Q_i x(t-d) \quad (15)$$

Then, It follows from (14) and (15) that

$$\dot{V}(x(t)) - \alpha V(x(t)) = \xi^T(t) \Delta_i \xi(t) \quad (16)$$

Where

$$\xi^T(t) = [x^T(t) \quad x^T(t-d) \quad \psi_1^T(i, t) \quad \psi_2^T(i, t)], \text{ and}$$

$$\begin{aligned} \psi_1(i, t) &= F_i(t) N_i x(t), \\ \psi_2(i, t) &= F_i(t) N_{di} x(t-d) \end{aligned} \quad (17)$$

$$\Delta_i = \begin{bmatrix} \varphi_{i,11} & P_i A_{di} & P_i M_i & P_i M_i \\ * & -e^{\alpha d} Q_i & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} \quad (18)$$

Also, If we prove that  $\Delta_i < 0$ , then we can indicate the USTDS is finite time stable in the following. Because of the existence of zero on the main diagonal matrix  $\Delta_i$ , we cannot simply conclude that  $\Delta_i < 0$ . We apply Lemma 2 to solve this problem which is caused by uncertainties. Using (4) and (17), we obtain

$$\begin{aligned} \psi_1^T(i, t) \psi_1(i, t) &= x^T(t) N_i^T F_i^T(t) F_i(t) N_i x(t) \leq x^T(t) N_i^T N_i x(t) \\ \psi_2^T(i, t) \psi_2(i, t) &= x^T(t-d) N_{di}^T F_i^T(t) F_i(t) N_{di} x(t-d) \leq \\ & x^T(t-d) N_{di}^T N_{di} x(t-d) \end{aligned} \quad (19)$$

According to Lemma 2, if

$$\dot{V}(x(t)) - \alpha V(x(t)) = \xi^T(t) \Delta_i \xi(t) < \Xi \quad (20)$$

Where

$$\begin{aligned} \Xi = & (\psi_1^T(i, t) \psi_1(i, t) - x^T(t) N_i^T N_i x(t)) + \\ & (\psi_2^T(i, t) \psi_2(i, t) - x^T(t-d) N_{di}^T N_{di} x(t-d)) \end{aligned} \quad (21)$$

Then, we will prove that system (1) is finite time stable.

Therefor (20) is rewritten such that

$$\dot{V}(x(t)) - \alpha V(x(t)) - \Xi < 0 \quad (22)$$

Now, using Lemma 2, we have

$$\pi_1|_{\varepsilon=1} = \Xi \leq 0 \quad (23)$$

Then  $\pi_0(x) = \dot{V}(x(t)) - \alpha V(x(t)) < 0$  holds if and only if

$$\pi_0(x) - \varepsilon \pi_1(x) < 0 \quad (24)$$

So, we find that (24) is equivalent to (22). Writing (22) in the matrix form, we have

$$\begin{bmatrix} H_{11} & H_{12} \\ * & H_{22} \end{bmatrix} < 0 \quad (25)$$

Where

$$\begin{aligned} H_{11} &= \begin{bmatrix} \varphi_{i,11} + N_i^T N_i & P_i A_{di} \\ * & -e^{\alpha d} Q_i + N_{di}^T N_{di} \end{bmatrix}, \\ H_{12} &= \begin{bmatrix} P_i M_i & P_i M_i \\ 0 & 0 \end{bmatrix}, H_{22} = \begin{bmatrix} -I & 0 \\ * & -I \end{bmatrix} \end{aligned}$$

According to the Lemma 1, (25) is equivalent to

$$\begin{bmatrix} \varphi_{i,11} & P_i A_{di} & N_i^T & 0 & P_i M_i & P_i M_i \\ * & -e^{\alpha d} Q_i & 0 & N_{di}^T & 0 & 0 \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \quad (26)$$

So, we have

$$V(x(t)) - \alpha V(x(t)) < 0 \quad (27)$$

According to (10) and (12), at the switching moment  $t_k$ , we will have

$$V(t) \leq \mu V(t_k^-) \quad (28)$$

So from (27) and (28) for  $t \in [t_k, t_{k+1})$  and using the iterative method

$$V(t) < e^{\alpha(t-t_k)} V(t_k) < \mu e^{\alpha(t-t_k)} V(t_k^-) < \mu e^{\alpha(t_k-t_{k-1})} V(t_{k-1}) < \dots < e^{\alpha(t-0)} \mu^{N_\sigma(0,t)} V(0) < e^{\alpha T_f} \mu^{N_\sigma(0,T_f)} V(0) \quad (29)$$

From definition 1, we know  $N_\sigma(0, T_f) < \frac{T_f}{\tau_a}$ , this leads to

$$V(t) < e^{\alpha T_f} \mu^{\frac{T_f}{\tau_a}} V(0) \quad (30)$$

Then

$$V(t) \geq x^T(t) P_i x(t) \geq \lambda_{\min}(\tilde{P}_i) x^T(t) R x(t) = \lambda_1 x^T(t) R x(t) \quad (31)$$

On the other hand

$$\begin{aligned} V(0) &\leq \lambda_{\max}(\tilde{P}_i) x^T(0) R x(0) \\ &+ d e^{\alpha d} \lambda_{\max}(\tilde{Q}_i) \sup_{-d \leq \theta \leq 0} \{x^T(\theta) R x(\theta)\} \leq \\ &(\lambda_2 + d e^{\alpha d} \lambda_3) \sup_{-d \leq \theta \leq 0} \{x^T(\theta) R x(\theta)\} \end{aligned} \quad (32)$$

Putting together (30)-(32) leads to

$$x^T(t) R x(t) \leq \frac{V(t)}{\theta_1} < \frac{e^{\alpha T_f} \mu^{\frac{T_f}{\tau_a}} [V(0)]}{\lambda_1} \leq \frac{(\lambda_2 + d e^{\alpha d} \lambda_3) c_1}{\lambda_1} e^{\alpha T_f} \mu^{\frac{T_f}{\tau_a}} \quad (33)$$

From (9), it follows that  $\ln(\lambda_1 c_2) - \alpha T_f - \ln[(\lambda_2 + d e^{\alpha d} \lambda_3) c_1] > 0$

By virtue of (11), we will have

$$\frac{T_f}{\tau_a} < \frac{\ln(\lambda_1 c_2 e^{-\alpha T_f}) - \ln[(\lambda_2 + d e^{\alpha d} \lambda_3) c_1]}{\ln \mu} \quad (34)$$

Substituting (34) into (33) leads to

$$x^T(t) R x(t) < c_2 \quad (35)$$

According to Definition 2, we know that system (1) with  $u(t)=0$  is finite time stable with respect to  $(c_1, c_2, T_f, R, \sigma(t))$ . The proof is completed at this point.

Remark 2. Parameter  $\mu$  in Theorem 1 can be selected as  $\mu = 1$ . In this case, we will have  $\tau_a > \tau_a^* = 0$  which denotes that

the switching signal  $\sigma$  can be arbitrary. It can be obtained from (10) that

$$P_i \leq P_j, \quad Q_i \leq Q_j, \quad \forall i, j \in L \quad (36)$$

It is possible to consider (36) in the form of (37):

$$P_j = P_i = P, \quad Q_j = Q_i = Q \quad (37)$$

This shows that a common Lyapunov-like function is needed for all subsystems such as

$$\begin{aligned} V(t) &= V_1(t) + V_2(t), \\ V_1(t) &= x^T(t) P x(t), \\ V_2(t) &= \int_{t-d}^t x^T(s) Q e^{\alpha(t-s)} x(s) \end{aligned} \quad (38)$$

Corollary 1. If there exist positive definite symmetric matrices  $P, Q$  with appropriate dimensions and positive scalar  $\alpha$ , such that

$$\begin{bmatrix} \varphi_{11} & P A_{di} & N_i^T & 0 & P M_i & P M_i \\ * & -e^{\alpha d} Q & 0 & N_{di}^T & 0 & 0 \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \quad (39)$$

$$(\lambda_{\max}(\tilde{P}) + d e^{\alpha d} \lambda_{\max}(\tilde{Q})) c_1 < c_2 \lambda_{\min}(\tilde{P}) e^{-\alpha T_f} \quad (40)$$

Then the unforced switched system (1) is uniformly finite time stable with respect to  $(c_1, c_2, T_f, R)$ , Where

$$\begin{aligned} \varphi_{11} &= A_i^T P + P A_i - \alpha P + Q, \\ \tilde{P} &= R^{-\frac{1}{2}} P R^{-\frac{1}{2}}, \quad \tilde{Q} = R^{-\frac{1}{2}} Q R^{-\frac{1}{2}}. \end{aligned}$$

Proof. Choose a common Lyapunov-like function as (38). The proof procedure is similar to that of Theorem (1), hence it is omitted.

The following theorem studies sufficient conditions for finite time stabilization of the USTDS (1) with state feedback controller.

### 3.2 Finite time stabilization analysis

Consider system (1), under the controller  $u(t) = K_{\sigma(t)} x(t)$ ,  $t \in (0, T_f)$ , the corresponding closed-loop system is given as follows

$$\begin{aligned} \dot{x}(t) &= (\hat{A}_{\sigma(t)} + \hat{B}_{\sigma(t)} K_{\sigma(t)}) x(t) + \hat{A}_{d\sigma(t)} x(t-d) \\ x(t) &= \varphi(t), \quad t \in [-d, 0] \end{aligned} \quad (41)$$

Theorem 2. Consider uncertain switched system (41). If for each  $i \in L$ , there exist positive definite symmetric matrices  $X_i, Y_i$  and matrix  $Z_i$  with appropriate dimensions and positive scalar  $\alpha, \mu \geq 1$  such that

$$\begin{bmatrix} \Lambda_{i,11} & A_{di} X_i & X_i N_i^T & 0 & Z_i^T N_{Bi}^T & M_i & M_i & M_i \\ * & -e^{\alpha d} Y_i & 0 & X_i N_{di}^T & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \quad (42)$$

$$\left[ \frac{1}{\lambda_{\min}(\tilde{X}_i)} + \frac{d e^{\alpha d}}{\lambda_{\min}(\tilde{X}_i \tilde{Y}_i^{-1} \tilde{X}_i)} \right] c_1 < \frac{c_2 e^{-\alpha T_f}}{\lambda_{\max}(\tilde{X}_i)} \quad (43)$$

$$X_i \leq \mu X_j, \quad X_i^{-1} Y_i X_i^{-1} \leq \mu X_j^{-1} Y_j X_j^{-1}, \quad \forall i, j \in L \quad (44)$$

Then, under the controller  $u(t) = K_{\sigma(t)}x(t)$  and the following average dwell time

$$\tau_a > \tau_a^* = \frac{T_f L n \mu}{\ln\left(\frac{c_2 e^{-\alpha T_f}}{\lambda_{\max}(\bar{X}_i)}\right) - \ln\left[\left(\frac{1}{\lambda_{\min}(\bar{X}_i)} + \frac{de^{\alpha d}}{\lambda_{\min}(\bar{X}_i \bar{Y}_i^{-1} \bar{X}_i)}\right) c_1\right]} \quad (45)$$

The corresponding closed-loop systems is finite time stable with respect to  $(c_1, c_2, T_f, R, \sigma(t))$ , where

$$\Lambda_{i,11} = X_i A_i^T + A_i X_i - \alpha X_i + Y_i + B_i Z_i + Z_i^T B_i^T, \quad \bar{X}_i = R^{\frac{1}{2}} X_i R^{\frac{1}{2}}, \quad \bar{Y}_i = R^{\frac{1}{2}} Y_i R^{\frac{1}{2}}$$

Moreover, the state feedback controller gain is given by  $K_i = Z_i X_i^{-1}$ .

**Proof.** Choose a Lyapunov-like function as Theorem 1. Taking the derivative of  $V(t)$  with respect to  $t$  along the trajectory of the USTDS (1) yields

$$\begin{aligned} \dot{V}(x(t)) - \alpha V(x(t)) &= x^T(t) (A_i^T P_i + P_i A_i - \alpha P_i + Q_i \\ &\quad + P_i B_i K_i + K_i^T B_i^T P_i) x(t) \\ &\quad + x^T(t) (M_i F_i(t) N_i)^T P_i x(t) \\ &\quad + x^T(t) P_i (M_i F_i(t) N_i) x(t) \\ &\quad + x^T(t) [(M_i F_i(t) N_{Bi} K_i)^T P_i \\ &\quad + P_i (M_i F_i(t) N_{Bi} K_i)] x(t) \\ &\quad + x^T(t-d) A_{di}^T P_i x(t) + x^T(t) P_i A_{di} x(t-d) \\ &\quad + x^T(t-d) (M_i F_i(t) N_{di})^T P_i x(t) \\ &\quad + x^T(t) P_i (M_i F_i(t) N_{di}) x(t-d) \\ &\quad + x^T(t-d) e^{\alpha d} Q_i x(t-d) \end{aligned} \quad (46)$$

Then, (46) is rewritten as the following linear inequality

$$\dot{V}(x(t)) - \alpha V(x(t)) = \bar{\xi}^T(t) \bar{\Delta}_i \bar{\xi}(t) \quad (47)$$

Where

$$\begin{aligned} \bar{\xi}^T(t) &= [x^T(t) \quad x^T(t-d) \quad \psi_1^T(i, t) \quad \psi_2^T(i, t) \quad \psi_3^T(i, t)], \\ \text{and} \\ \psi_1(i, t) &= F_i(t) N_i x(t), \\ \psi_2(i, t) &= F_i(t) N_{di} x(t-d), \\ \psi_3(i, t) &= [F_i(t) N_{Bi} K_i] x(t) \end{aligned} \quad (48)$$

$$\bar{\Delta}_i = \begin{bmatrix} \bar{\varphi}_{i,11} & P_i A_{di} & P_i M_i & P_i M_i & P_i M_i \\ * & -e^{\alpha d} Q_i & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix} \quad (49)$$

$$\bar{\varphi}_{i,11} = A_i^T P_i + P_i A_i - \alpha P_i + Q_i + P_i B_i K_i + K_i^T B_i^T P_i$$

Now, If we prove that  $\bar{\Delta}_i < 0$ , then we can show the USTDS is finite time stabilization in the following.. Using (4) and (48), we obtain

$$\psi_1^T(i, t) \psi_1(i, t) = x^T(t) N_i^T F_i^T(t) F_i(t) N_i x(t) \leq x^T(t) N_i^T N_i x(t)$$

$$\begin{aligned} \psi_2^T(i, t) \psi_2(i, t) &= x^T(t-d) N_{di}^T F_i^T(t) F_i(t) N_{di} x(t-d) \leq \\ & x^T(t-d) N_{di}^T N_{di} x(t-d) \quad \psi_3^T(i, t) \psi_3(i, t) = \\ & x^T(t) K_i^T N_{Bi}^T F_i^T(t) F_i(t) N_{Bi} K_i x(t) \leq x^T(t) K_i^T N_{Bi}^T N_{Bi} K_i x(t) \end{aligned} \quad (50)$$

Like the proof of Theorem 1 (20)-(24), according to Lemma2, we have

$$\begin{bmatrix} \bar{H}_{11} & \bar{H}_{12} \\ * & \bar{H}_{22} \end{bmatrix} < 0 \quad (51)$$

Where

$$\begin{aligned} \bar{H}_{11} &= \begin{bmatrix} \bar{\varphi}_{i,11} + N_i^T N_i + K_i^T N_{Bi}^T N_{Bi} K_i & P_i A_{di} \\ * & -e^{\alpha d} Q_i + N_{di}^T N_{di} \end{bmatrix}, \\ \bar{H}_{12} &= \begin{bmatrix} P_i M_i & P_i M_i & P_i M_i \\ 0 & 0 & 0 \end{bmatrix}, \\ \bar{H}_{22} &= \begin{bmatrix} -I & 0 & 0 \\ * & -I & 0 \\ * & * & -I \end{bmatrix} \end{aligned}$$

According to the Lemma 1, (51) is equivalent to

$$\begin{bmatrix} \bar{\varphi}_{i,11} & P_i A_{di} & N_i^T & 0 & K_i^T N_{Bi}^T & P_i M_i & P_i M_i & P_i M_i \\ * & -e^{\alpha d} Q_i & 0 & N_{di}^T & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \quad (52)$$

Using  $\text{diag}\{P_i^{-1}, P_i^{-1}, I, I, I, I, I, I\}$  to pre and post-multiply the left term of (52), and let  $X_i = P_i^{-1} > 0$ ,  $Y_i = P_i^{-1} Q_i P_i^{-1} > 0$ , we have

$$\begin{bmatrix} \Gamma_{i,11} & A_{di} X_i & X_i N_i^T & 0 & X_i K_i^T N_{Bi}^T & M_i & M_i & M_i \\ * & -e^{\alpha d} Y_i & 0 & X_i N_{di}^T & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \quad (53)$$

$$\Gamma_{i,11} = X_i A_i^T + A_i X_i - \alpha X_i + Y_i + B_i K_i X_i + X_i K_i^T B_i^T$$

Let  $K_i = Z_i X_i^{-1}$ , then (53) is equivalent to (42). From (9)-(11), for  $P_i = X_i^{-1}$  and  $Q_i = X_i^{-1} Y_i X_i^{-1}$ , we have condition (43)-(45). The proof is completed at this point.

We will obtain the following stability conditions in the matrix form with analyzing nonlinear condition (43) and (45).

**Corollary 2.** Consider the USTDS (41). If for each  $i \in L$ , there exist positive definite symmetric matrices  $X_i, Y_i$  and matrix  $Z_i$  with appropriate dimensions and positive scalars  $\alpha, \lambda_1, \lambda_2, \lambda_3, \mu \geq 1$  such that the conditions (9), (42), and

$$\begin{bmatrix} \bar{X}_i & I \\ I & \lambda_2 I \end{bmatrix} > 0 \quad (54)$$

$$\begin{bmatrix} \lambda_1^{-1} I & I \\ I & \bar{X}_i^{-1} \end{bmatrix} > 0, \quad \begin{bmatrix} \bar{Y}_i^{-1} & \bar{X}_i^{-1} \\ \bar{X}_i^{-1} & \lambda_3 I \end{bmatrix} < 0 \quad (55)$$

Then, under the average dwell time (11), the closed-loop systems (41) is finite time stable with respect to

$(c_1, c_2, T_f, R, \sigma(t))$ . The state feedback controller gain is given by  $K_i = Z_i X_i^{-1}$ .

Proof. Let

$$\lambda_1 I < \tilde{X}_i^{-1}, \lambda_2 I > \tilde{X}_i^{-1}, \lambda_3 I > \tilde{X}_i^{-1} \tilde{Y}_i \tilde{X}_i^{-1} \quad (56)$$

Then from (43) and (56) we obtain (9). Furthermore from (45) and (56) we will get (11). According to the Schure complement, from (56) we obtain (54)-(55). The proof is complete at this point.

**Remark 3.** It is worth noting that in Corollary 2 the inequalities are not in the LMIs form due to (55). To solve this non-convex feasibility problem, we use the following minimization algorithm subject to LMI constraints [43].

*USTDS problem*

$$\min \{ \lambda_1 \gamma + \text{trace} \sum_{i \in I} (\tilde{X}_i \mathcal{R}_i + \tilde{Y}_i \mathcal{S}_i) \}$$

Subject to (9), (42), (54) and

$$\begin{aligned} \begin{bmatrix} \gamma I & I \\ I & \mathcal{R}_i \end{bmatrix} > 0, \begin{bmatrix} \mathcal{S}_i & \mathcal{R}_i \\ \mathcal{R}_i & \theta_3 I \end{bmatrix} > 0, \begin{bmatrix} \tilde{X}_i & I \\ I & \mathcal{R}_i \end{bmatrix} \geq 0, \begin{bmatrix} \tilde{Y}_i & I \\ I & \mathcal{S}_i \end{bmatrix} \geq 0, \\ 0, \begin{bmatrix} \lambda_1 & 1 \\ 1 & \gamma \end{bmatrix} \geq 0 \quad \forall i \in L \end{aligned} \quad (57)$$

If the solution of the above minimization problem is equal to  $2n+1$ , then the conditions in Corollary 2 are solvable. The algorithm in detail is developed below that  $n$  and  $k$  denote state variables and the number of iterations, respectively.

*USTDS algorithm*

*Step 1.* Find a feasible set  $(\tilde{X}_i^0, \tilde{Y}_i^0, \mathcal{R}_i^0, \mathcal{S}_i^0, \lambda_1^0, \lambda_2^0, \lambda_3^0, \gamma^0)$  satisfying (9), (42), (54) and (57). Set  $k = 0$ .

*Step 2.* Solve the following minimization problem

$$\begin{aligned} \mathbb{T}^* = \min \{ \lambda_1^k \gamma + \lambda_1 \gamma^k \\ + \text{trace} \sum_{i \in I} (\tilde{X}_i^k \mathcal{R}_i + \tilde{Y}_i^k \mathcal{S}_i + \tilde{X}_i \mathcal{R}_i^k \\ + \tilde{Y}_i \mathcal{S}_i^k) \} \end{aligned}$$

Subject to (9), (42), (54), (57) and denotes  $\mathbb{T}^*$  be the optimized value.

*Step 3.* If the matrix inequalities (9), (42), (54) and (57) are satisfied and

$$\left| \lambda_1^k \gamma + \lambda_1 \gamma^k + \text{trace} \sum_{i \in I} (\tilde{X}_i^k \mathcal{R}_i + \tilde{Y}_i^k \mathcal{S}_i + \tilde{X}_i \mathcal{R}_i^k + \tilde{Y}_i \mathcal{S}_i^k) - (2n+1) \right| < \delta$$

Holds for a sufficiently small scalar  $\delta > 0$ , then  $(\tilde{X}_i, \tilde{Y}_i, \mathcal{R}_i, \mathcal{S}_i, \lambda_1, \lambda_2, \lambda_3, \gamma)$  are a feasible solution and exit.

*Step 4.* If  $k > q$ , where  $q$  is the maximum number of iteration allowed, then exit. Otherwise, set  $k = k + 1$  and go to Step 2.

#### 4. Numerical examples

Now, two examples are employed to verify the proposed theorem in this paper.

**Example 1.** Consider the uncertain open loop switched system (1) with parameters as

$$\begin{aligned} A_1 &= \begin{bmatrix} -1.7 & 1.7 & 0 \\ 1.3 & -1 & 0.7 \\ 0.7 & 1 & -0.6 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & -1 & 0 \\ 0.7 & 0 & -0.6 \\ 1.7 & 0 & -1.7 \end{bmatrix} \\ A_{d1} &= \begin{bmatrix} 1.5 & -1.7 & 0.1 \\ -1.3 & 1 & -0.3 \\ -0.7 & 1 & 0.6 \end{bmatrix}, A_{d2} = \begin{bmatrix} -1 & 0 & 0.1 \\ 1.3 & -0.1 & 0.6 \\ 1.5 & 0.1 & 1.8 \end{bmatrix} \\ N_1 &= \begin{bmatrix} 0.2 & 0 & -0.7 \\ 0 & -0.9 & -0.1 \\ -0.5 & -0.1 & 0 \end{bmatrix}, N_2 = \begin{bmatrix} -0.8 & 0 & -0.7 \\ 0 & 0.4 & 0.1 \\ -0.5 & 0 & -0.8 \end{bmatrix} \\ N_{d1} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, N_{d2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ M_1 &= \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}, M_2 = \begin{bmatrix} 0.02 & 0 & 0 \\ 0 & 0.02 & 0 \\ 0 & 0 & 0.02 \end{bmatrix} \\ \phi(t) &= [0.7 \ 0 \ 0]^T \end{aligned}$$

The values of  $c_1, c_2, T_f, d$  and matrix  $R$  given as follows

$$c_1 = 0.5, c_2 = 100, T_f = 10, d = 0.2, R = I$$

By virtue of Theorem 1 and solving (8) and (9) for  $\alpha = 0.015$  leads to feasible solutions

$$\begin{aligned} P_1 &= \begin{bmatrix} 65.3342 & 8.3026 & -11.0567 \\ 8.3026 & 95.2520 & -48.1515 \\ -11.0567 & -48.1515 & 70.9817 \end{bmatrix}, \\ P_2 &= \begin{bmatrix} 101.5078 & 6.3782 & 52.0920 \\ 6.3782 & 41.3732 & 2.0302 \\ 52.0920 & 2.0302 & 73.7269 \end{bmatrix}, \\ Q_1 &= \begin{bmatrix} 30.4514 & -32.1724 & 44.9440 \\ -32.1724 & 81.9195 & -36.8890 \\ 44.9440 & -36.8890 & 102.2389 \end{bmatrix}, \\ Q_2 &= \begin{bmatrix} 30.5022 & -32.1381 & 44.9399 \\ -32.1381 & 82.5640 & -36.4247 \\ 44.9399 & -36.4247 & 102.5921 \end{bmatrix} \end{aligned}$$

According to (11), one obtains  $\tau_a > \tau_a^* = 4.1308$ . Then, by using of theorem 1 for any switching signal  $\sigma_1(t)$  with average dwell time  $\tau_a > \tau_a^*$  switched system (1) with  $u(t) = 0$  is finite time stable with respect to  $(c_1, c_2, T_f, R, \sigma_1)$ . We choose  $\tau_a = 4.15$ . The phase plot of state and the norm of the state vector for the open loop switched system are shown in Figure 1 and Figure 2. It is clear that the unforced switched system (1) is finite time stable under switching signal  $\sigma_1$ . For guaranteeing finite time stable of the switched systems (1), we need the switching signal is slow switching. If the switching is very frequent, it is possible that the system is not finite time stable. The switching signal and the norm of the state vector of the unforced USTDS under a periodic switching signal  $\sigma_2(t)$  over  $0 \sim 10$  with average dwell time  $\tau_a = 1.2$  are shown in Figure 3 and Figure 4. It is observed that the unforced switched system (1) is not FTS with respect to  $(c_1, c_2, T_f, R, \sigma_2)$ .

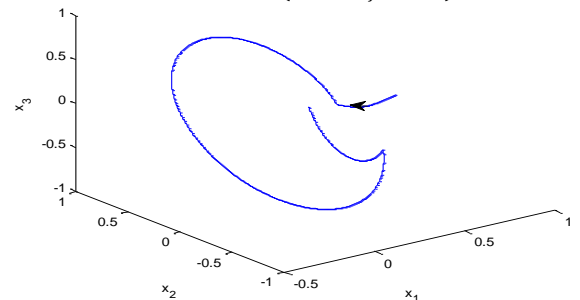
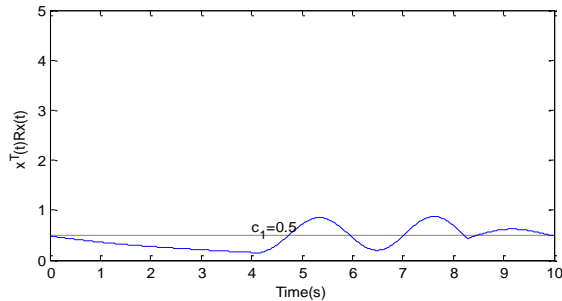
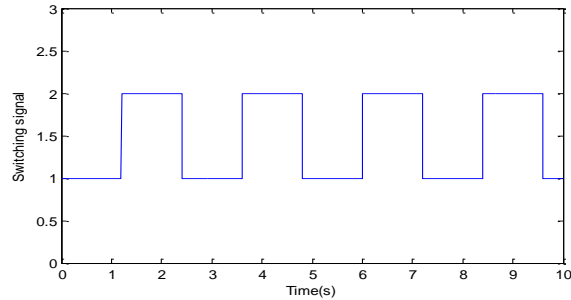
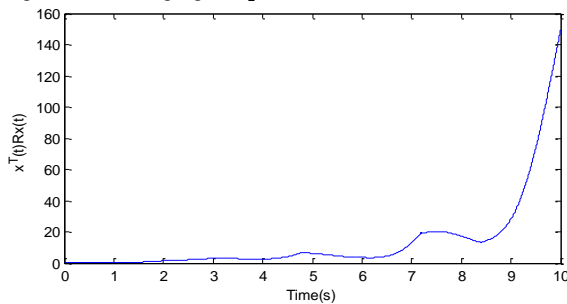


Figure 1. Phase plot of state  $x(t)$

Figure 2. Time history of  $x^T(t)Rx(t)$  under switching signal  $\sigma_1$ Figure 3. Switching signal  $\sigma_2$ Figure 4. Time history of  $x^T(t)Rx(t)$  under switching signal  $\sigma_2$ 

Example 2. Consider the USTDS (41) with parameters of example1 and as follows

$$N_{B1} = \begin{bmatrix} 0.02 & 0 & 0 \\ 0 & 0.02 & 0 \\ 0 & 0 & 0.02 \end{bmatrix},$$

$$N_{B2} = \begin{bmatrix} -0.03 & 0 & 0 \\ 0 & -0.03 & 0 \\ 0 & 0 & -0.03 \end{bmatrix},$$

$$B1 = \begin{bmatrix} 1 & 0.1 & 0.5 \\ 0.6 & 1 & 0.4 \\ 0.3 & 0.2 & 1 \end{bmatrix}, \quad B2 = \begin{bmatrix} -1 & 0.5 & 0.1 \\ 0.2 & 0 & 0.6 \\ 1 & 0.4 & -0.1 \end{bmatrix}$$

The values of  $c_1, c_2, T_f, d$  and matrix  $R$  given as follows

$$c_1 = 0.5, \quad c_2 = 50, \quad T_f = 10, \quad d = 0.2, \quad R = I$$

From Corollary 2 and the CCL algorithm, we get the matrix solutions for  $\alpha = 0.015$  as follows

$$X_1 = \begin{bmatrix} 1.0551 & 0.2686 & -0.0365 \\ 0.2686 & 0.9546 & 0.0538 \\ -0.0365 & 0.0538 & 1.2671 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} 0.9483 & 0.0135 & -0.3264 \\ 0.0135 & 1.4340 & -0.0215 \\ -0.3264 & -0.0215 & 1.0164 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} 2.8644 & -0.0645 & 0.0295 \\ -0.0645 & 2.7466 & -0.0258 \\ 0.0295 & -0.0258 & 2.7807 \end{bmatrix},$$

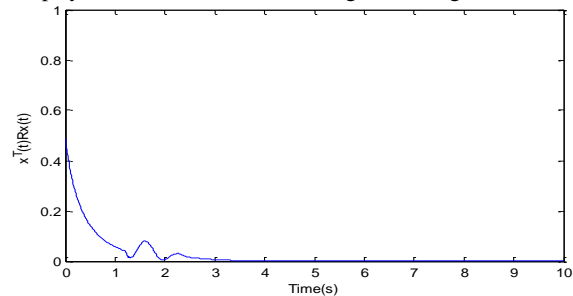
$$Y_2 = \begin{bmatrix} 2.8017 & -0.0036 & -0.1822 \\ -0.0036 & 2.9479 & 0.0071 \\ -0.1822 & 0.0071 & 2.7906 \end{bmatrix}$$

The corresponding state feedback matrices are:

$$K_1 = \begin{bmatrix} -1.7671 & 0.1983 & 0.3196 \\ 0.2403 & -1.9954 & -0.3909 \\ 0.5794 & -0.7111 & -1.6402 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 3.5379 & -1.8038 & 2.7650 \\ -4.5434 & -3.9339 & -5.4013 \\ 4.9042 & -2.6474 & 8.9467 \end{bmatrix}$$

According to (11), we get  $\tau_a > \tau_a^* = 1.1805$ . We choose  $\tau_a = 1.2$ , hence the closed loop system is robust finite time stable with respect to  $(c_1, c_2, T_f, R, \sigma_2)$ . The norm of the state vector of the closed loop system with state feedback is given in Figure 5.

Figure 5. Time history of  $x^T(t)Rx(t)$  of closed loop system

## 5. Conclusion

In this paper, robust finite time stability and stabilization problems for a class of switched systems with time delay have been investigated. The uncertainties under consideration are norm bounded and time varying in the model. Bases on the average dwell time method and multiple Lyapunov-like functions, sufficient conditions which can guarantee finite time stability and stabilization of the USTDS are presented. The state feedback controller design problem is solved by using the cone complement linearization algorithm. The problem of uncertainty in switched systems with time delay is investigated by virtue of Yakubovich lemma. In most of literature, FTS and FTSz of STDS are discussed with time-dependent switching. It is needed that switching sequence to be known in advance and fast switching is not allowed with time-dependent switching, but state-dependent switching is based on the current value of the system states, that is more practical and the switching sequence does not require to be known beforehand. A challenging further investigation is how to extend the results in this paper with state-dependent switching.

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