Design of the Model Predictive Controller Based on Orthonormal Basis Functions for Automotive Air Conditioning System

Pegah Khavash, Amin Ramezani*, and Sadjaad Ozgoli

***Abstract*— Air conditioning system (A/C) of the car imposes an additional load on the engine, increasing fuel consumption and losses. Therefore, any improvement in its performance has a direct impact on vehicle performance and fuel consumption. The automotive A/C system is a Multi Input- Multi Output (MIMO) plant and There are constraints on its variables So the method of Model Predictive Control (MPC) as an effective method. So far the MPC method is implemented largely for this system. In this paper a predictive control method based on orthonormal functions is provided for automotive air conditioning system. System's model has been changed with an embedded integrator, inputs and outputs changes are highly penalized in cost function and Laguerre orthonormal basis functions are added in MPC's structure and it will be shown that in the proposed control method compared to the conventional MPC method, the automotive air conditioning system performance has been improved and because of reduced computational load the runtime of simulations implementation has been reduced.**

***Index Terms*—Automotive air conditioning system, Model Predictive Controller, Orthonormal functions**

I. INTRODUCTION

AN air conditioning (A/C) system is often identified as a system operated on the Vapor Compression Cycle (VCC). As illustrated in Fig. *1*, this cycle consists of an evaporator, a condenser, a compressor, an expansion valve and a fan. The refrigerant enters the compressor as a slightly superheated vapor at a low pressure. Afterwards it leaves the compressor and enters the condenser as a vapor at an exalted pressure, where the refrigerant is condensed as heat is transitioned to the outside environment. The refrigerant then leaves the condenser as a high-pressure liquid. The pressure of the liquid decreases by going through the expansion valve. Consequently, some of the liquid beams into cold vapor. The rest of the liquid, now at a low pressure and temperature, is vaporized in the evaporator while heat is transferred from the refrigerated space. This vapor then re-enters the compressor [1].

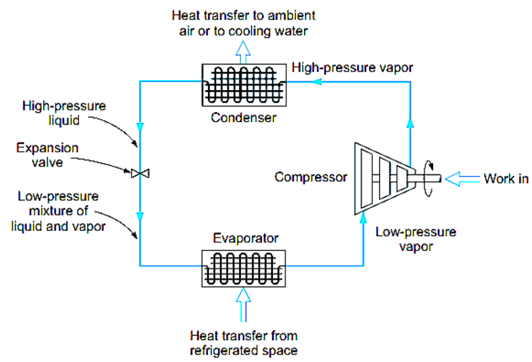

Fig. 1. Schematic diagram of a refrigeration cycle [1].

In cabins or buildings among all energy consuming factors, cooling by the A/C system plays an important role. Normally the on/off operation of these systems are implemented to rich to the desired environment which consumes significant power. Recently, modern air conditioners have begun to consolidate variable speed and variable-position actuators to improve energy efficiency and cooling performance. Moreover, solar air-conditioning systems emerge and begin to be utilized in reality. Utilizing solar energy system is a promising mean of both reducing consumption of fossil fuels and CO_2 emissions into the atmosphere. Regardless of the kind of A/C system, a crucial ingredient to accomplish good performance and efficient energy consumption is a proper control strategy [2].

Various methods and control approaches are applied on air conditioning system yet. For example, in reference number [3], a decoupled proportional-integral (PI) control with pre-compensator is presented. However, according to the studies carried on by Lin and Yeh, there are strong cross-couplings among inputs and outputs. They have improved feedback control algorithms which had been incorporated with a

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traditional PI controller. The applied feedback controller is multi-input and multi-output-based and owns a cascade structure for dealing with the fast and slow dynamics in the system [4]. In reference number [5], Zhang et al. Recommended an application of sliding mode control to an automotive A/C system aiming at regulating the superheat temperature and cooling capacity. Recently, they have demonstrated an energy-optimal control for ancillary load reduction of this system [6]. One of the controllers which widely uses in industry, is Model Predictive controller (MPC). One of the significant aspects of MPC which makes the design methodology applicable to both practitioners and academics is the ability of the method in handling both 'soft' constraints and hard constraints in a multivariable control framework. On the other hand, MPC can execute on-line process optimization. Another advantage of this method is the simplicity of the design framework in dealing with all these complex issues [7]. As mentioned earlier, the A/C system is a multivariable plant which has strong cross-couplings among its inputs and outputs. There are different constraints in the system such as input saturation limits and state or output constraints. Therefore, model predictive control is noted as a convenient approach to control the system due to its features. Razi et al. in reference number [8] proposed a neuro-predictive controller for temperature control of automotive A/C system. For this purpose, an adaptive neural predictive control is proposed in [9]. Leducq et al. implemented MPC to control a VCC using a first principle non-linear model of this cycle [10]. An MPC method has been introduced to control the evaporator superheat and condensing pressure in [11]. In [12] a predictive control scheme is designed to control a transport refrigeration system such as a delivery truck in which a VCC configured in parallel to a thermal energy storage unit is included. Wallace et al. used the data generated from a first principle model of a VCC to identify a linear model, then they designed an offset-free model predictive controller based on this model [13]. Recently, in [14] another utilization of offset free MPC is implemented for an energy efficient operation of the central chiller plant in a case study hotel on a tropical island.

As it is clear, MPC is a widely utilized means of A/C systems and chilling plants, but here an advanced MPC is designed to develop performance of the automotive A/C system. The model of system is changed with an imbedded integrator, inputs and outputs changes are highly penalized in cost function and Laguerre orthonormal basis functions are added in MPC's structure. It will be depicted that the energy saving and cooling capacity of automotive A/C system are improved towards regular MPC while decreasing computational load and simulations runtime.

II. Automotive A/C System Model

Most system oriented models of A/C systems are today based on Moving Boundary Method (MBM), which is a parameter – limited modeling technique that could be utilized in heat exchangers to model dynamics related to fluid's mass and heat transportation regarding their (fluids) phase changes. The A/C system models resulted from MBM are generally in the form of high order nonlinear differential and algebraic equations [5]. In reference number [5], Zhang et al. has proposed an MBM model of automotive A/C system that is expressed in a matrix form as:

$$Z(x)\frac{dx}{dt} = f(x,u)$$
$$y = g(x) \tag{1}$$

The complete A/C system model is characterized by 15 states, which result from applying mass and energy balances to the condenser and evaporator. This model is more difficult even for local controller design. Hence, they presented a 6-states linear model of the system through a model order reduction approach and data driven method. Inputs are compressor rotation speed in the unit of rpm and expansion valve opening percentage, $[N_c \quad \alpha]^T$. The outputs are the superheat temperature at the evaporator (SH) and the pressure difference between the condenser and evaporator (Δp). Superheating refrigerant beyond the evaporation temperature is important since not superheating means that the refrigerant enters as in two-phase into the compressor and increase the power consumption and wear. It is also important to have as much two-phase refrigerant in the evaporator as possible to increase the heat transfer and consequently optimize the refrigeration process. Therefore, a crucial variable which significantly affects the efficiency of a cooling system is the superheat [15]. From another point of view, when a passenger requires lower temperature in the section or a solar sensor detects sunshine increase, the supervisory temperature controller either increases blower fan speed or makes the refrigeration controller lower the evaporation pressure in order to increase the amount of heat exchange in the evaporator. In these situations, remaining unchanged for the superheat can boost heat exchange efficiency at the evaporator and can result in saving energy as well [3].

In this paper, the specified model is used to adjust superheat temperature and cooling capacity of automotive A/C system. Working point of the model is characterized by a compressor speed of 3000rpm and an expansion valve opening of 23.3%. Reciprocal nominal outputs are 5oC superheat and 1090kPa pressure difference.

III. MPC Using Laguerre Functions

In this paper, conventional MPC is replaced with orthonormal based MPC in order to improve the performance of automotive A/C.

In mentioned method tuning is simpler compared to conventional MPC due to more free parameters. Applying orthonormal functions reduces number of parameters utilized for description of future control trajectory, and computational volume decreases as a result. Finally, penalizing input and output changes provokes smoother responses in this method.

A. *Augmented Model*

Assume that a discrete time model comes as following:

$$x_m(k+1) = A_m x_m(k) + B_m u(k)$$
$$y(k) = C_m x_m(k) \tag{2}$$

Assume that the plant has p inputs, q outputs and n1 states. We need to change the model to be suitable for our design purpose in which an integrator is embedded. The augmented model can be expressed as follows according to [7]:

$$\overbrace{\begin{bmatrix} \Delta x_m(k+1) \\ y(k+1) \end{bmatrix}}^{x(k+1)} = \overbrace{\begin{bmatrix} A_m & O_m^T \\ C_m A_m & I_{q\times q} \end{bmatrix}}^{A} \overbrace{\begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix}}^{x(k)} + \overbrace{\begin{bmatrix} B_m \\ C_m B_m \end{bmatrix}}^{B} \Delta u(k) \quad (3)$$

$$y(k) = \overbrace{\begin{bmatrix} O_m & I_{q\times q} \end{bmatrix}}^{C} \begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix}$$

Where $I_{q\times q}$ is the identity matrix with dimensions $q \times q$, which is the number of outputs; and O_m is a $q \times n1$ zero matrix. In (3), A_m, B_m and C_m have dimension $n1 \times n1$, $n1 \times m$ and $q \times n1$, respectively. Where $\Delta u(k) = u(k) - u(k-1)$ and $\Delta x_m(k) = x_m(k) - x_m(k-1)$ denote the difference of the control input and the state variable respectively.

B. Introducing the Laguerre Functions

The z-transfer function of Leaguers function is given as [7]

$$\Gamma_k(z) = \Gamma_{k-1}(z)\frac{z^{-1}-a}{1-az^{-1}} \quad (4)$$

With $\Gamma_1(z) = \frac{\sqrt{1-a^2}}{1-az^{-1}}$ where $0 \le a < 1$ is called the scaling factor and is picked by the user. Letting $l_1(k)$ to $l_N(k)$ denote the inverse z-transforms of $\Gamma_1(z)$ to $\Gamma_N(z)$. This set of discrete-time Laguerre functions are expressed in a vector form as:

$$L_k = [l_1(k) \quad l_2(k) \quad \dots \quad l_N(k)]^T \quad (5)$$

Regarding (4), the set of discrete-time Laguerre functions in vector (5) fits in the following equation:

$$L(k+1) = A_l L(k) \quad (6)$$

In which matrix A_l is $(N \times N)$ and is a function of parameters a and $\beta = (1-a^2)$, and the initial condition is represented by

$$L(0)^T = \sqrt{\beta}[1 \quad -a \quad \dots \quad (-1)^{N-1}a^{N-1}] \quad (7)$$

C. Use of Laguerre Functions in MPC Design

In MIMO predictive control system setting, each input signal is dedicated to have a Laguerre pole location independently. Let

$$\Delta u = [\Delta u_1(k) \quad \Delta u_2(k) \quad \dots \quad \Delta u_p(k)]^T \quad (8)$$

and the input matrix be partitioned to

$$B = B_1\, B_2 \dots B_p \quad (9)$$

We declare the ith control signal Δu_i(k) by choosing a scaling factor a_i and order N_i, where a_i and N_i are picked for this particular input, such that

$$\Delta u_i(k) = \sum_{j=1}^{N_i} c_j^i(k) l_j^i(k) \quad (10)$$

By considering $\eta_i = \begin{bmatrix} c_1^i & c_2^i & \dots & c_{N_i}^i \end{bmatrix}$ and $L_i(k)^T = \begin{bmatrix} l_1^i(k), l_2^i(k), \dots, l_{N_i}^i(k) \end{bmatrix}$,

$$\Delta u_i(k) = L_i(k)^T \eta_i \quad (11)$$

where η_i and L_i(k) are the Laguerre network description of the ith control.

The state prediction has the following form:

$$\begin{aligned} x(k_i+m|k_i) &= A^m x(k_i) \\ &+ \sum_{j=0}^{m-1} A^{m-j-1}[B_1 L_1(j)^T \quad B_2 L_2(j)^T \quad \dots \quad B_m L_m(j)^T] \\ &= A^m x(k_i) + \phi(m)^T \eta \end{aligned} \quad (12)$$

Where the parameter vector η and the data matrix $\varphi(m)^T$ include individual coefficient vectors given by

$$\eta = \begin{bmatrix} \eta_1^T & \eta_2^T & \dots & \eta_p^T \end{bmatrix} \quad (13)$$

$$\phi(m)^T = \sum_{j=0}^{m-1} A^{m-j-1}[B_1 L_1(j)^T \quad B_2 L_2(j)^T \quad \dots \quad B_m L_m(j)^T] \quad (14)$$

The cost function could be mentioned in the following quadratic form

$$J = \sum_{m=1}^{N_p} (x(k_i+m|k_i))^T \times R_y \times (x(k_i+m|k_i)) + \sum_{m=0}^{N_p-1} (\Delta U(k_i+m|k_i))^T \times R_u \times (\Delta U(k_i+m|k_i)) \quad (15)$$

The weighting matrices are $R_y > 0$ and $R_u > 0$. By substituting (11) into the cost function (15) we obtain

$$J = \eta^T \Omega \eta + 2\eta^T \psi x(k_i) + \sum_{m=1}^{k_p} x(k_i)^T (A^T)^m R_y A^m x(k_i) \quad (16)$$

where the matrices Ω and Ψ are

$$\Omega = (\sum_{m=1}^{N_p} \phi(m) Q \phi(m)^T + R_L$$

$$\psi = (\sum_{m=1}^{N_p} \phi(m) Q A^m) \quad (17)$$

Consequent to achieving the optimal parameter vector η in existence of inputs and states constraints, the receding horizon control law would be realized as

$$\Delta u(k_i) = \begin{bmatrix} L_1(0)^T & 0_2^T & \dots & 0_m^T \\ 0_1^T & L_2(0)^T & \dots & 0_m^T \\ \vdots & \vdots & \ddots & \vdots \\ 0_1^T & 0_2^T & \dots & L_m(0)^T \end{bmatrix} \eta \quad (18)$$

Where 0_k^T, $k = 1, 2, \dots m$ demonstrates a zero block row vector with identical dimension to $L_k(0)^T$.

D. Laguerre MPC in Presence of Constraints

Suppose that the limits on the control signals are u_{min} and u_{max}. Noting that the increment of the control signal is $u(k) = \sum_{i=0}^{k-1} \Delta u(i)$, then the inequality constraint for the future time k, $k = 1, 2, \dots$, is expressed as :

$$u_{min} \le$$

$$\begin{bmatrix} \sum_{i=0}^{k-1} L_1(i)^T & 0_2^T & \cdots & 0_m^T \\ 0_1^T & \sum_{i=0}^{k-1} L_2(i)^T & \cdots & 0_m^T \\ \vdots & \vdots & \ddots & \vdots \\ 0_1^T & 0_2^T & \cdots & \sum_{i=0}^{k-1} L_m(i)^T \end{bmatrix} \eta + u(k_i - 1) \leq u_{max} \quad (19)$$

IV. SIMULATION

In this paper, the used control strategies are discrete time methods. The model of automotive A/C system that is introduced in reference number [5] is a continuous time model. Since this model will be the basis of controller design of present paper, it must be discrete with proper sample time. The largest aigen value of the model is 509.33 so the smallest time constant is $\mathrm{T_s} = (1/509.33)$ second. 20 percent of $\mathrm{T_s}$ is considered as discretization time.

In present paper, three cases have been simulated. First and second cases are implementation of regular MPC method, and third case is the result of Laguerre MPC simulation. In the regular MPC, there are parameters such as N_p and N_c which the designer can bring the system to appropriate behavior. So, tow simulations have been done with the different prediction and control horizons; [N_p=170, N_c=10] and [N_p=200, N_c=30] in case 1 and case2, respectively.

As mentioned before, tuning is simple in MPC based on orthonormal basis functions, because of more free parameters such as: the poles of Laguerre network, $= [a_1 \quad a_2]$, and the number of terms making the Laguerre network, $N = [N_1 \quad N_2]$. A scaling factor a_i and order N_i are selected for ith input. The designer must choose proper N_i for each a_i such that system represents desired treatment. In third simulation (case 3), Laguerre MPC method is implemented with $N = [20 \quad 20]$ and $a = [0.4 \quad 0.4]$. By the way, prediction and control horizones are same with the values of Np and Nc in case 1. The tracking of setpoints by the outputs, and control efforts of the manipulated variables are illustrated in Fig. 2 and Fig. 3, respectively.

In the whole simulations, constraints applied on manipulated variables (u_{min}, u_{max}), the inputs and outputs weight matrices ($\mathrm{R_u}$, $\mathrm{R_y}$) are set with the same values which are given in TABLE . By the way, the results of both methods have been done by a laptop with 4GB and core i5 processor, and runtime for each of simulations are reported in TABLE .

TABLE I
THE VALUES OF CONSTRAINTS AND WEIGHT MATRICES

u_{min}	$\begin{bmatrix} 2000 \\ 10 \end{bmatrix}$
u_{max}	$\begin{bmatrix} 4000 \\ 40 \end{bmatrix}$
R_u	$\begin{bmatrix} 0.0001 & 0 \\ 0 & 0.0001 \end{bmatrix}$
R_y	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

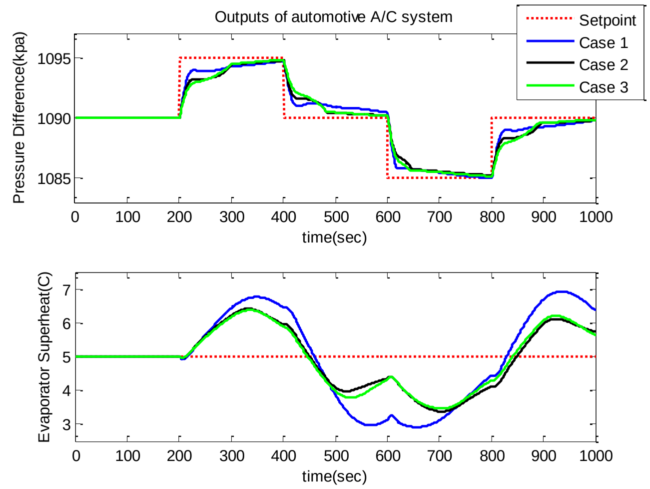

Fig. 2. Outputs of the automotive A/C system via MPC (with two different horizons) and Laguerre based MPC

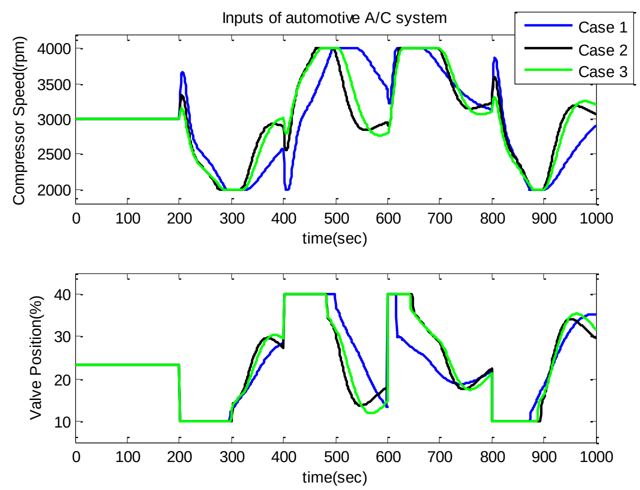

Fig. 3. Inputs of the automotive A/C system via MPC (with two different horizons) and Laguerre based MPC

TABLE II
RUNTIME OF SIMULATIONS

Simulation	Runtime (ms)
Case 1	**5070**
Case 2	**21758**
Case 3	9942

As shown in Fig. 2 and Fig. 3, the comparison of case 1 and case 2 illustrates that by the increase of prediction and control horizons pressure difference between the condenser and evaporator, better tracking of the reference signal is achieved. Besides that, overshoot and undershoot of superheat temperature is reduced. Also, this change in parameters in case 2 leads to lower control efforts of compressor and expansion valve rather than case 1. However, computational load and simulations runtime is increased according to TABLE . The simulation results in case 3 are the same as case 2, while the runtime is less because of using the Laguerre functions in the proposed control structure. Thus Laguerre MPC has shown its superiority by showing better performance while its computational load and runtime of simulations are less than regular MPC.

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